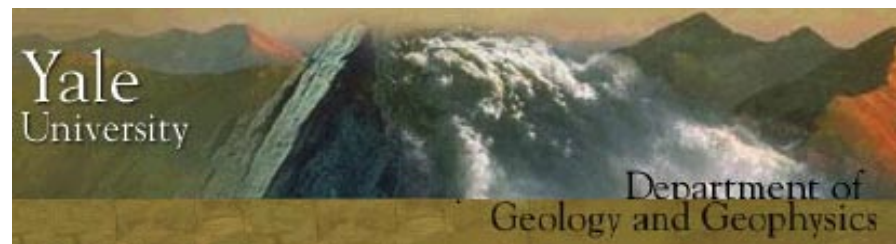


AEROSOLS AND CLIMATE CHANGE CERTAINTIES AND UNCERTAINTIES

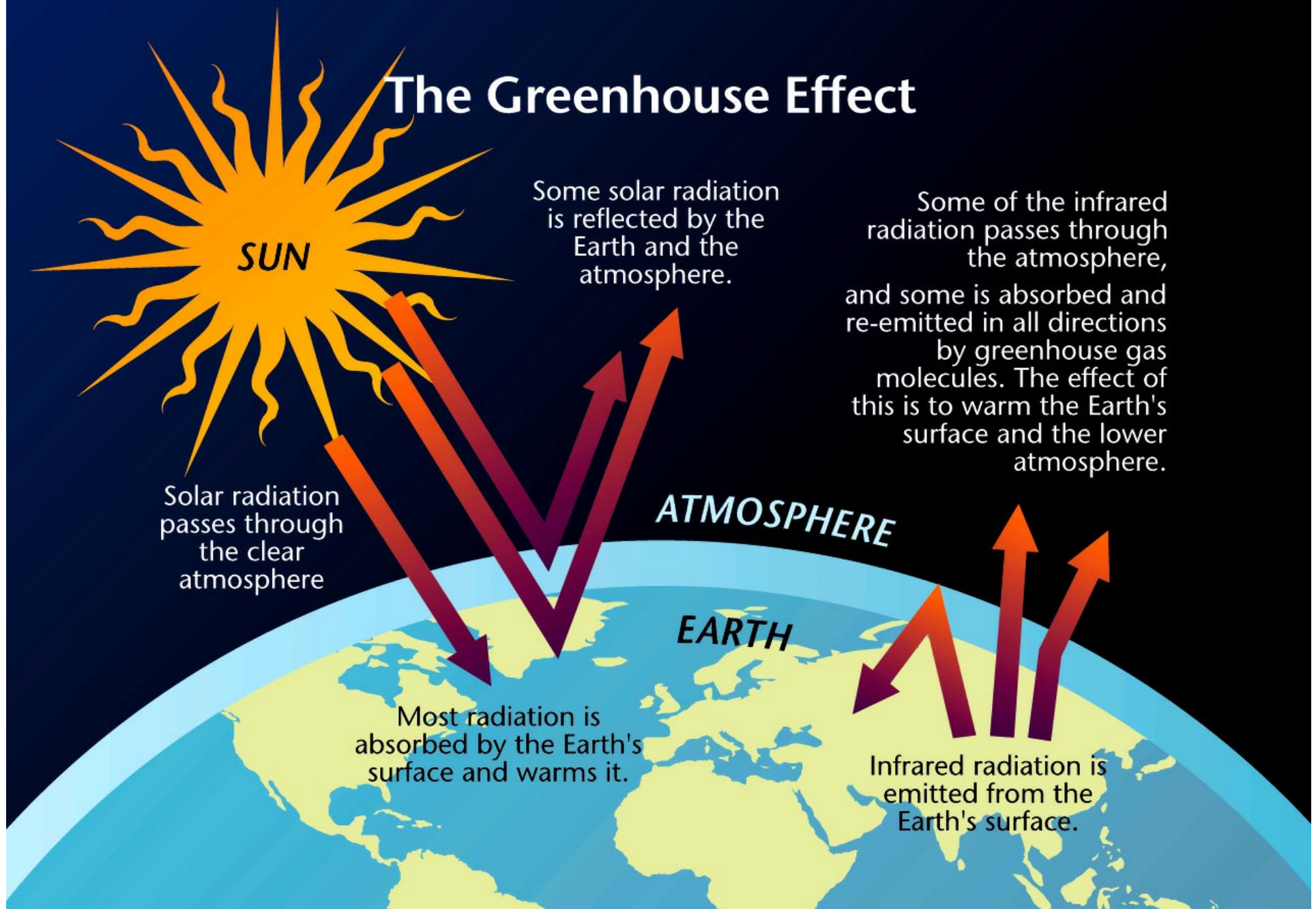
Stephen E. Schwartz



March 1, 2004

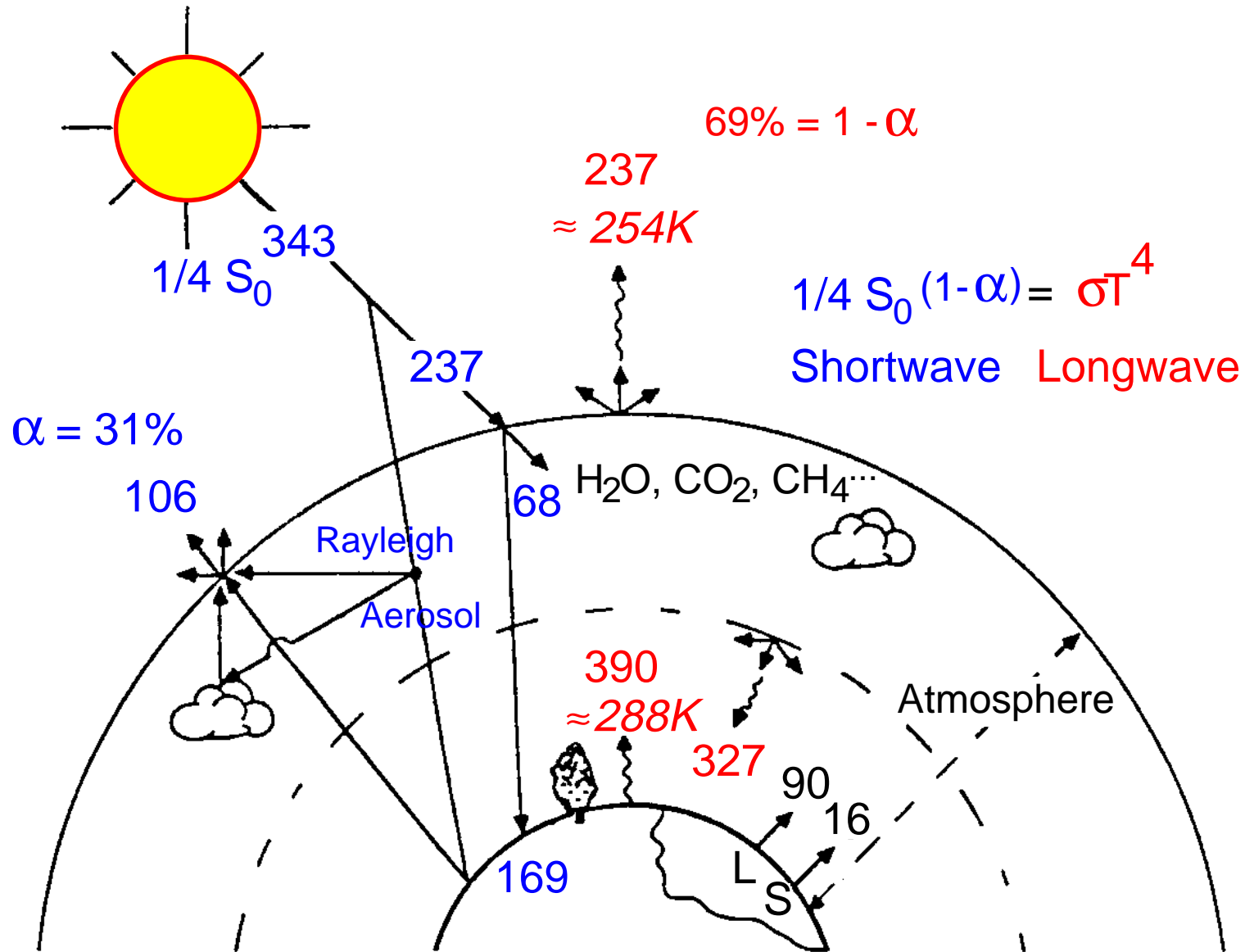
<http://www.ecd.bnl.gov/steve/schwartz.html>

The Greenhouse Effect



GLOBAL ENERGY BALANCE

Global and annual average energy fluxes in watts per square meter



Schwartz, 1996, modified from Ramanathan, 1987

ATMOSPHERIC RADIATION

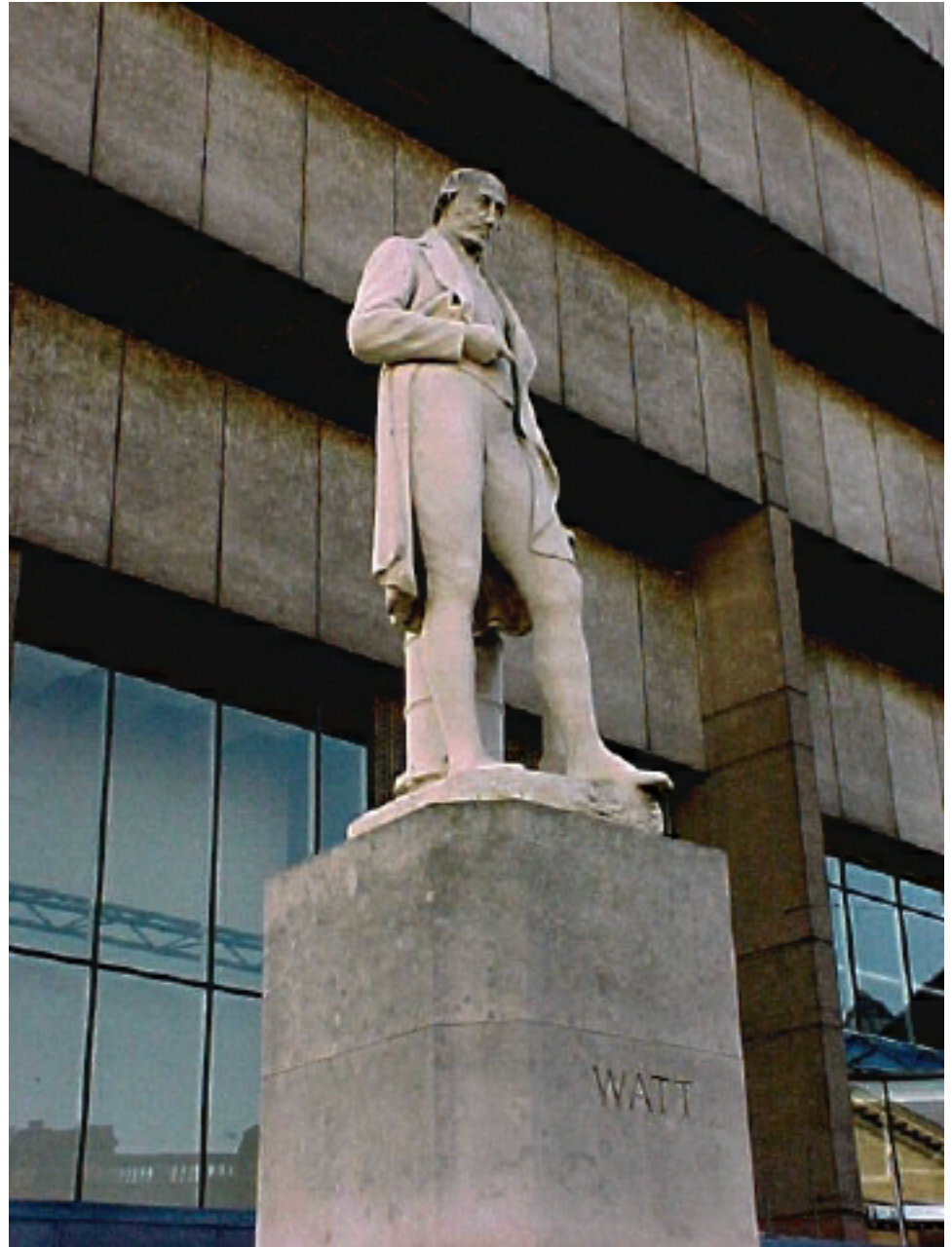
*Energy per area per
time*

Power per area

Unit:

Watt per square meter

$W m^{-2}$



RADIATIVE FORCING OF CLIMATE CHANGE

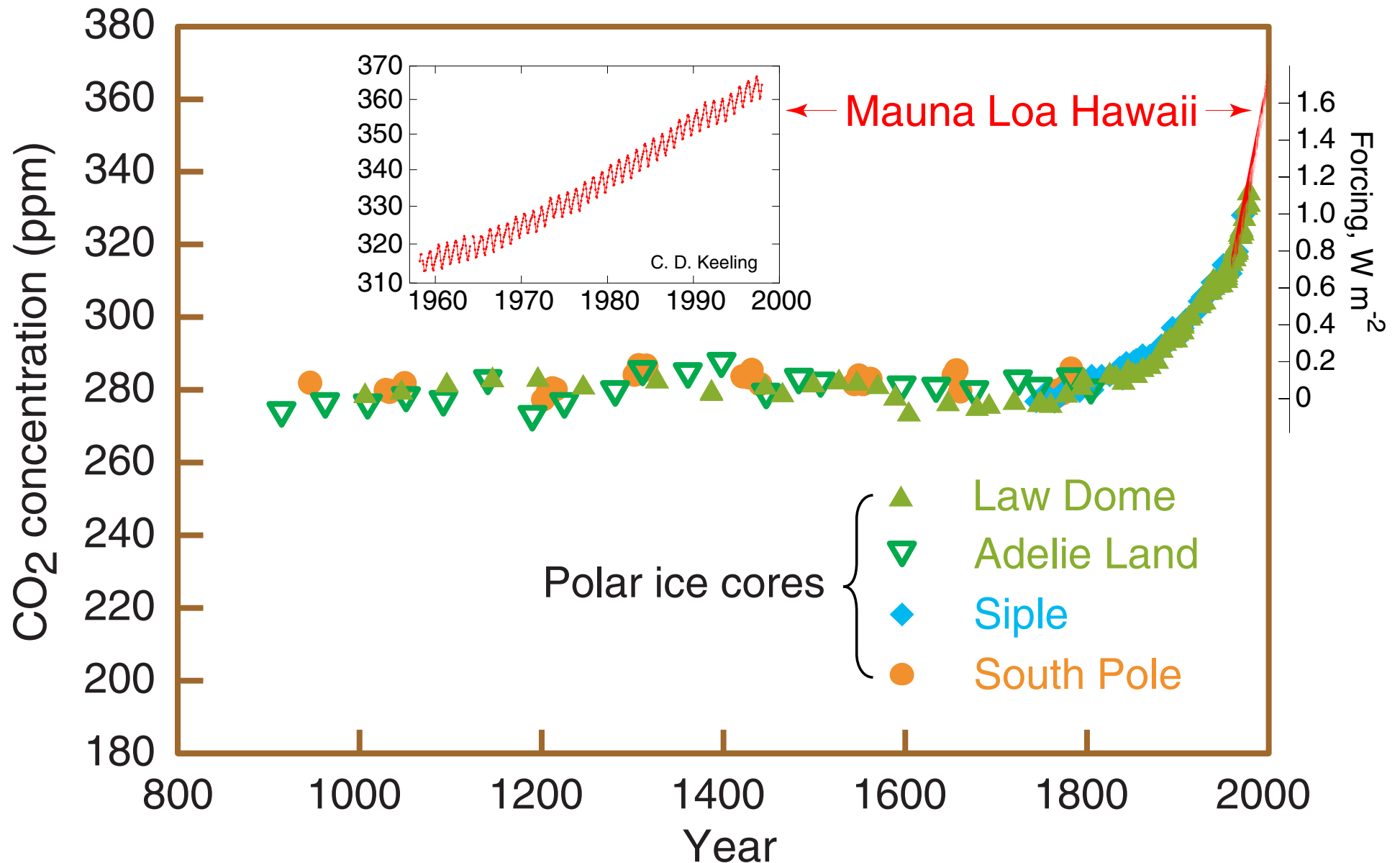
A ***change*** in a radiative flux term in Earth's radiation budget, F , W m^{-2} .

Working hypothesis:

On a global basis radiative forcings are additive and fungible.

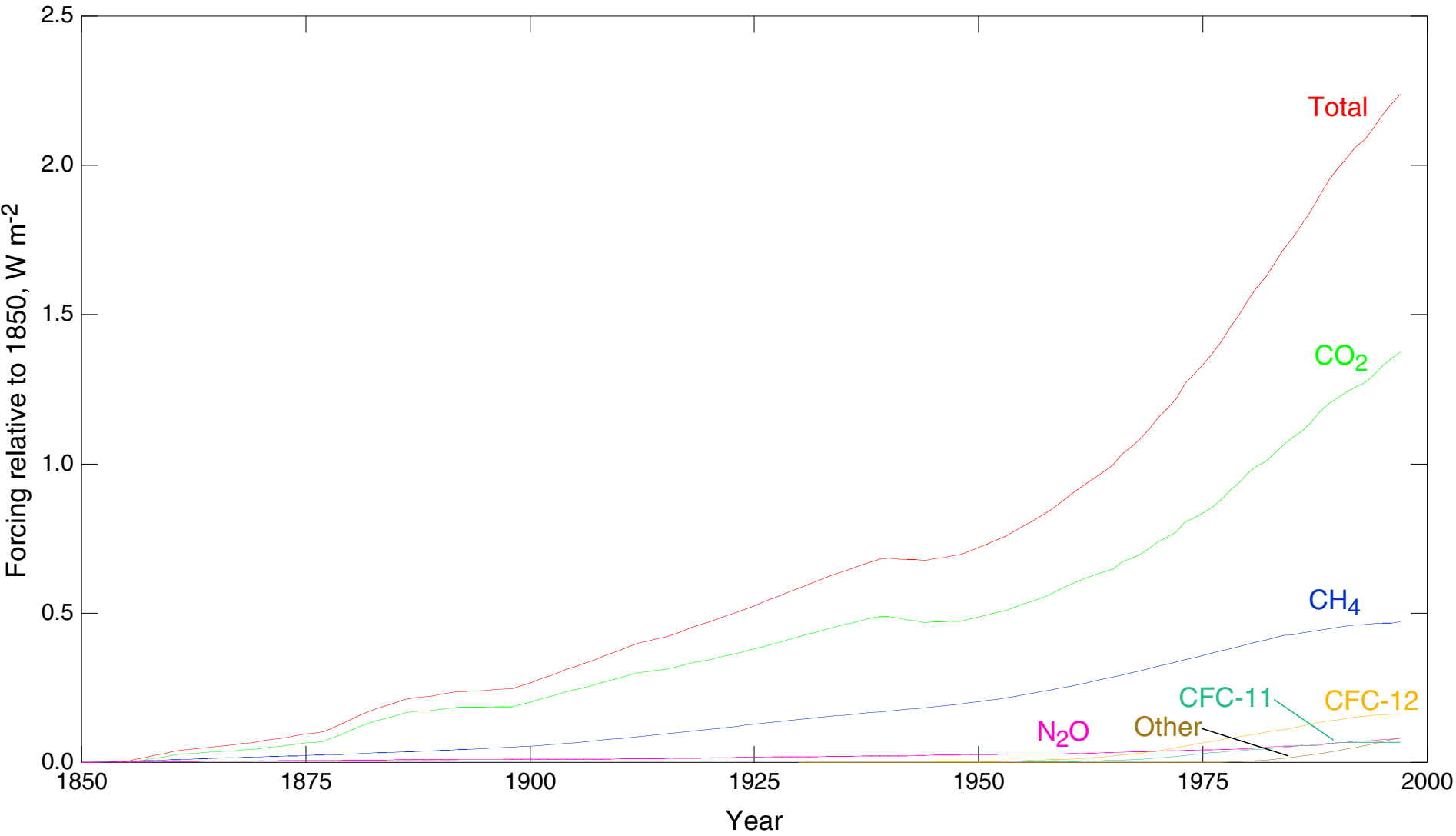
- This hypothesis is fundamental to the radiative forcing concept.
- This hypothesis underlies much of the assessment of climate change over the industrial period.

ATMOSPHERIC CARBON DIOXIDE IS INCREASING



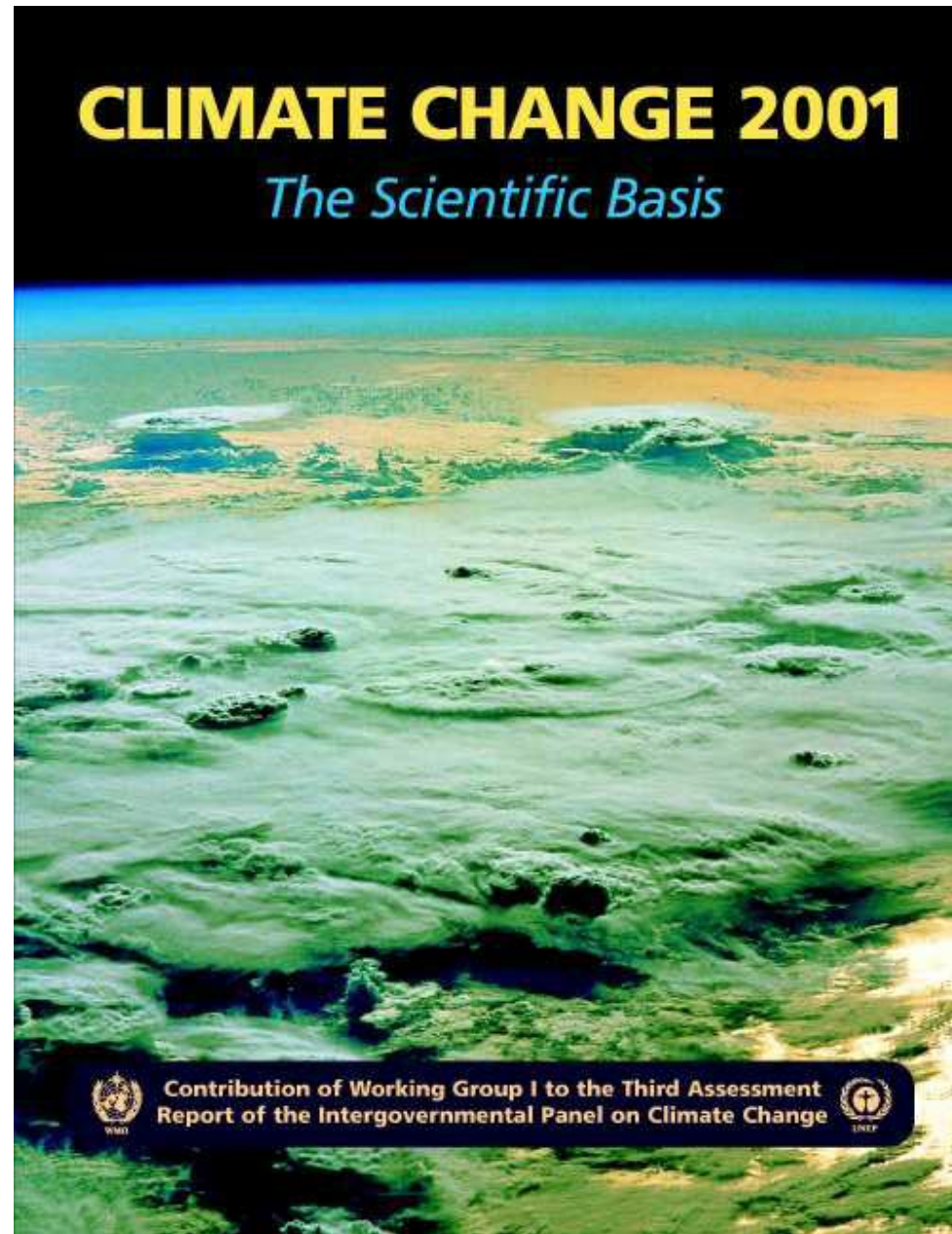
Global carbon dioxide concentration and infrared radiative forcing over the last thousand years

GREENHOUSE GAS FORCINGS OVER THE INDUSTRIAL PERIOD



Data: GISS

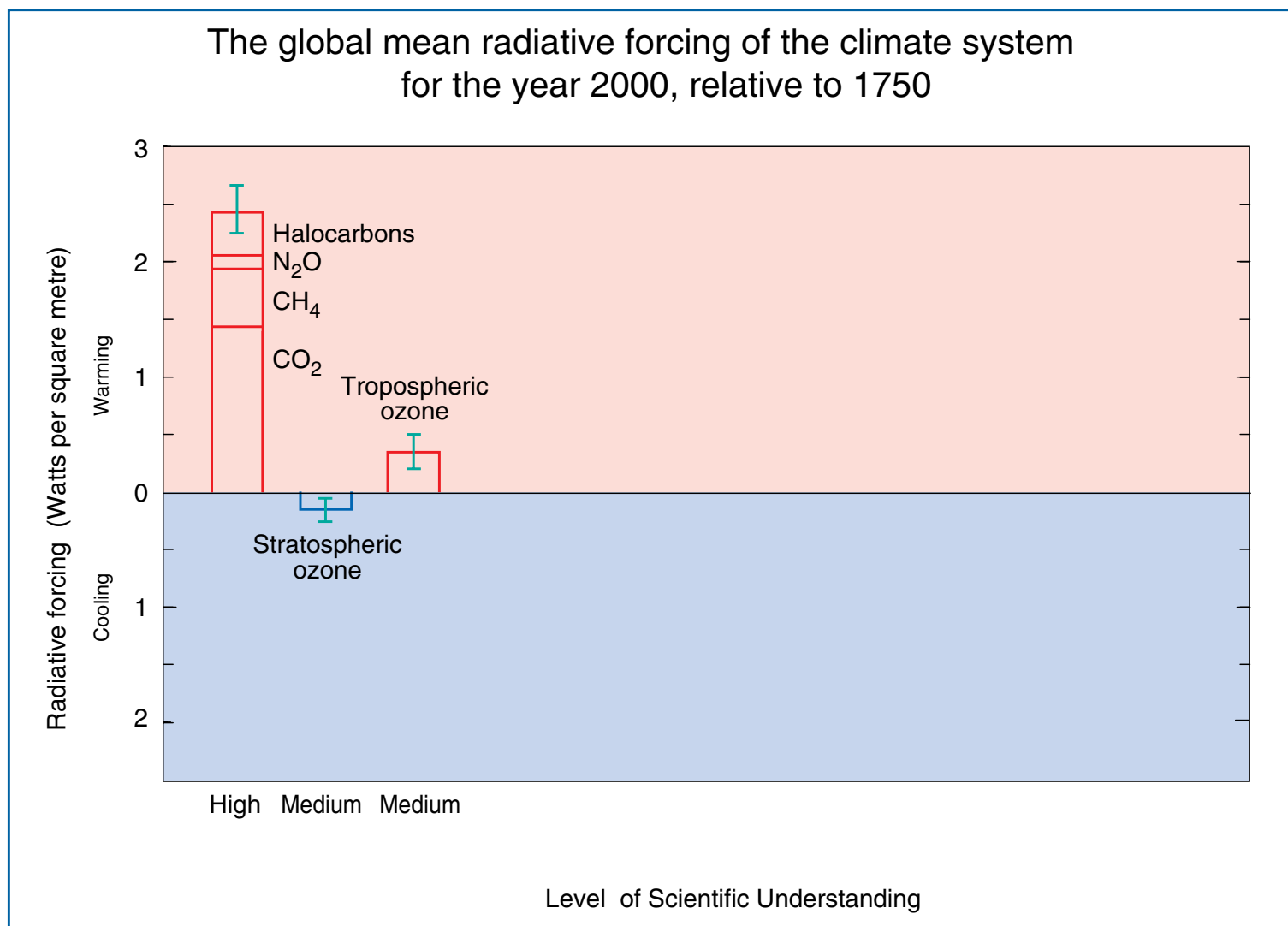
THE “BIBLE” OF CLIMATE CHANGE RESEARCH



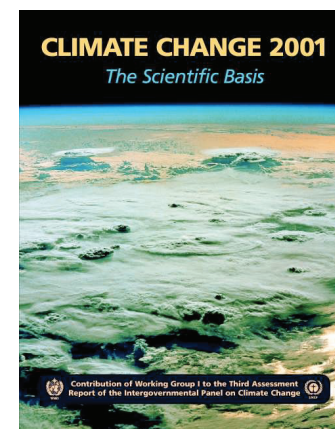
Cambridge University Press, 2001

RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD IPCC (2001)

Greenhouse gases only



Summary for Policymakers A Report of Working Group I of the Intergovernmental Panel on Climate Change



CLIMATE RESPONSE

The ***change*** in global and annual mean temperature, ΔT , K, resulting from a given radiative forcing.

Working hypothesis:

The change in global mean temperature depends on the magnitude of the forcing, not its nature or its spatial distribution.

$$\Delta T = \lambda F$$

CLIMATE SENSITIVITY

The ***change*** in global and annual mean temperature per unit forcing, λ , K/(W m⁻²).

TOP-LEVEL QUESTION IN CLIMATE CHANGE SCIENCE

- *How much will the global mean temperature change?*

$$\Delta T = \lambda F$$

where F is the *forcing* and λ is the *climate sensitivity*.

- A *forcing* is a change in a radiative flux component, W m^{-2} .
- Forcings are thought to be *additive* and *fungible*.

- *What is Earth's climate sensitivity?*

- *National Academy Report (Charney, 1979):*

$$F = 4 \text{ W m}^{-2}$$

“ We estimate the most probable global warming for a doubling of CO₂ to be *near 3 degrees C*, with a probable error of *plus or minus 1.5 degrees*.

- *Intergovernmental Panel on Climate Change (IPCC, 2001):*

“ Climate sensitivity [to CO₂ doubling] is likely to be in the range *1.5 to 4.5°C*.

This uncertainty is not very useful for policy planning.

HOW CAN CLIMATE SENSITIVITY BE DETERMINED?

$$\text{Climate sensitivity } \lambda = \Delta T / F$$

- *Climate models* evaluated by performance on prior climate change, and/or
- *Empirical determination* from prior climate change.
- Either way, ΔT and F must be determined with sufficiently small uncertainty to yield an uncertainty in λ that is useful for informed decision making.

CLIMATE CHANGE SENSITIVITY

Summary of 15 Current Models

Quantity, Unit	Mean	Standard Deviation	Range
λ , K/(W m ⁻²)	0.87	0.23	0.5 - 1.25
$\Delta T_{2\times}$, K	3.5	0.9	2 - 5

IPCC *Climate Change 2001*, Cambridge University Press, 2001

EMPIRICAL CLIMATE SENSITIVITY

Greenhouse forcing over the industrial period is 2.5 W m^{-2}

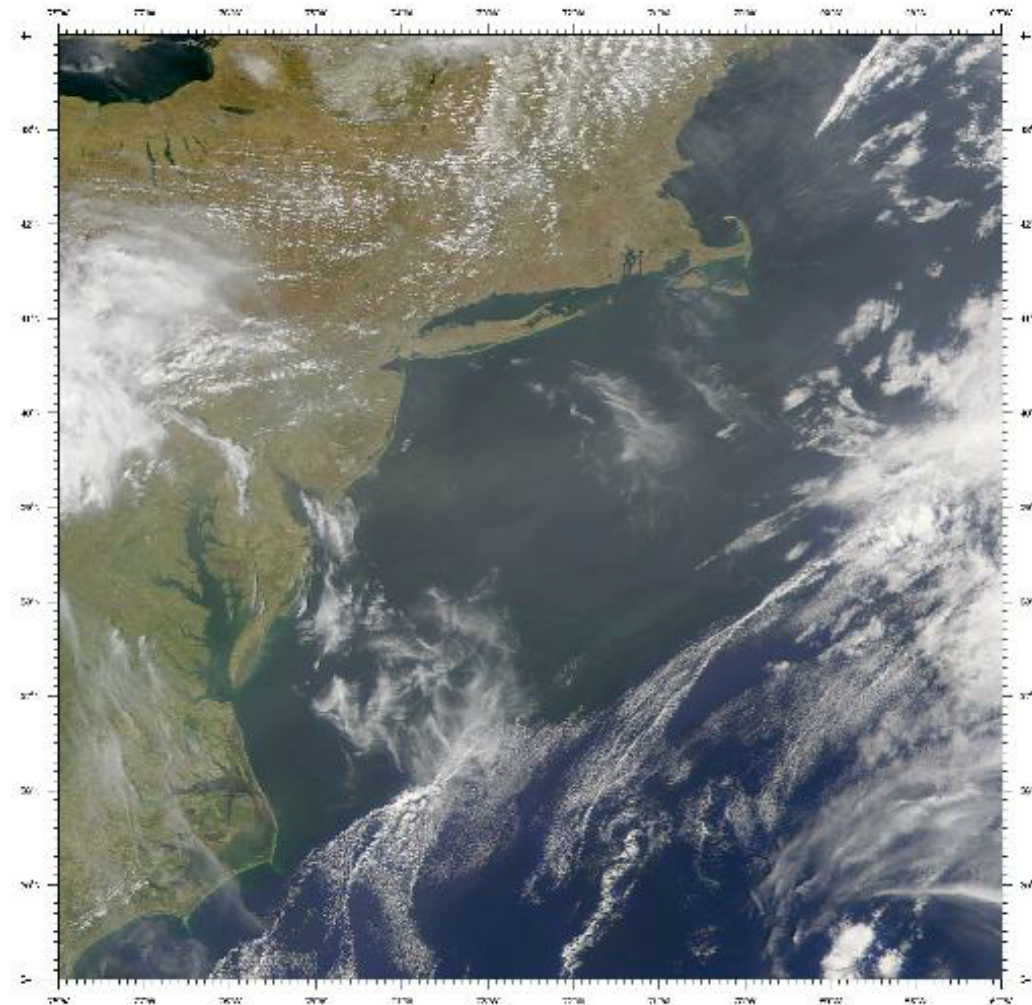
Temperature increase over the industrial period is 0.6 K .

Empirical Sensitivity:

$$\lambda = \frac{dT}{dF} = \frac{0.6 \text{ K}}{2.5 \text{ W m}^{-2}} = 0.24 \text{ K / (W m}^{-2}\text{)} \quad \text{or} \quad \Delta T_{2\times} = 1 \text{ K}$$

Why is the empirical sensitivity so much lower than model-based estimates?

AEROSOL: A suspension of particles in air

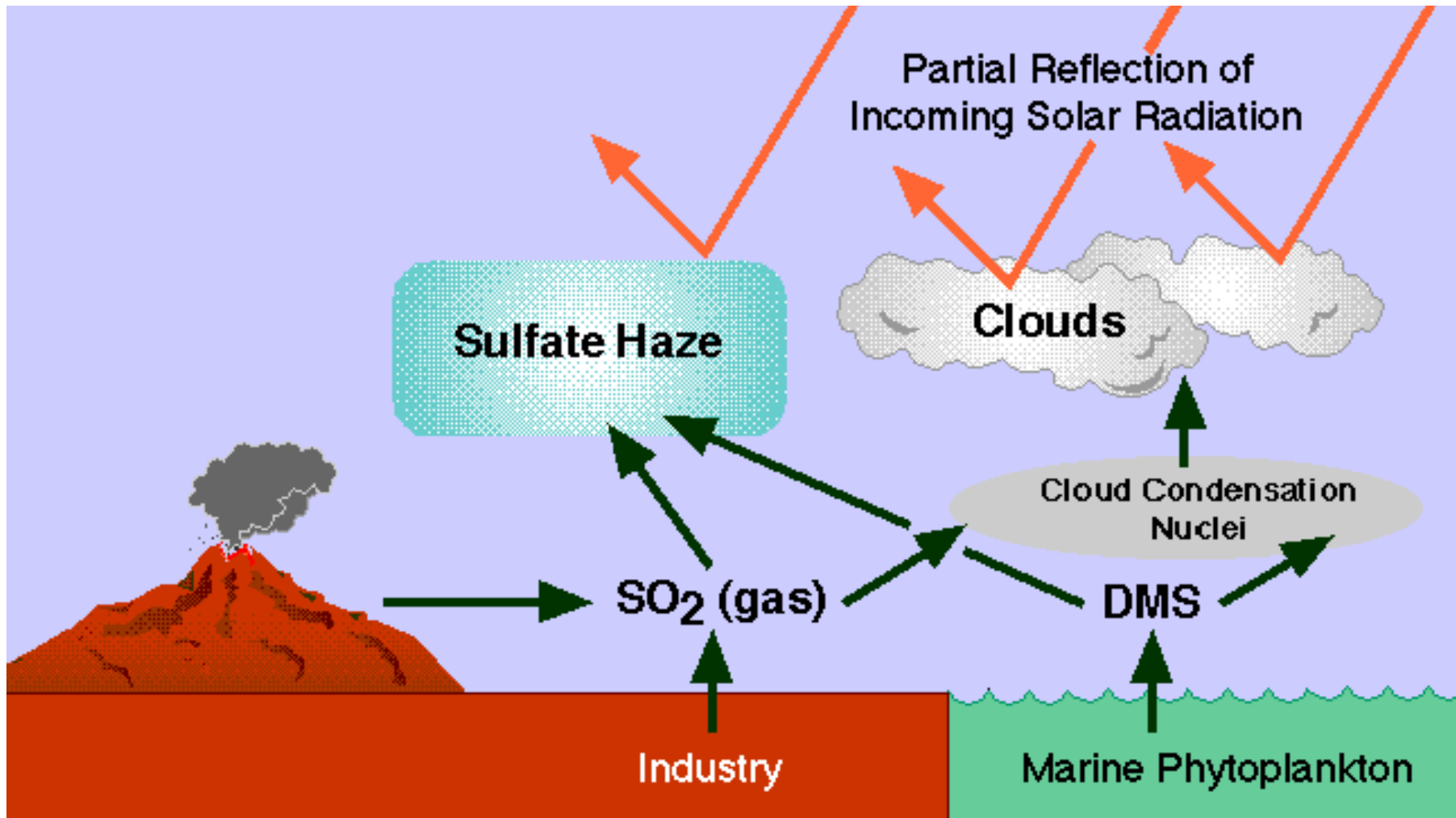


2001-04-22-17:28

SeaWiFS Project, NASA/Goddard Space Flight Center, and ORBIMAGE

Atmospheric aerosols may result from primary emissions (dust, smoke) or from gas to particle conversion in the atmosphere (haze, smog).

RADIATIVE FORCING OF CLIMATE CHANGE BY AEROSOLS



AEROSOL INFLUENCES ON RADIATION BUDGET AND CLIMATE

Direct Effect (Cloud-free sky)

Light scattering -- Cooling influence

Light absorption -- Warming influence, depending on surface

Indirect Effects (Aerosols influence cloud properties)

More droplets -- Brighter clouds (Twomey)

More droplets -- Enhanced cloud lifetime (Albrecht)

Semi-Direct Effect

Absorbing aerosol heats air and evaporates clouds

DIRECT EFFECT

BIOMASS BURNING AND WIDESPREAD AEROSOL

Northeastern Oklahoma, 2000-12-01



DIRECT RADIATIVE FORCING DUE TO ANTHROPOGENIC SULFATE AEROSOL

$$\overline{\Delta F_R} = -\frac{1}{2} F_T T^2 (1 - A_c)(1 - R_s)^2 \cdot \overline{\beta} \alpha_{\text{SO}_4^{2-}} f(\text{RH}) \cdot \underbrace{Q_{\text{SO}_2} Y_{\text{SO}_4^{2-}} \left(\frac{\text{MW}_{\text{SO}_4^{2-}}}{\text{MW}_S} \right) \tau_{\text{SO}_4^{2-}}}_{\text{Column Burden Atmospheric Chemistry}} \bigg/ A$$

Geophysics
Aerosol Optical Depth
Aerosol Microphysics

$\overline{\Delta F_R}$ is the area-average shortwave radiative forcing due to the aerosol, W m^{-2}

F_T is the solar constant, W m^{-2}

A_c is the fractional cloud cover

T is the fraction of incident light transmitted by the atmosphere above the aerosol

R_s is the albedo of the underlying surface

$\overline{\beta}$ is upward fraction of the radiation scattered by the aerosol,

$\alpha_{\text{SO}_4^{2-}}$ is the scattering efficiency of **sulfate and associated cations** at a reference low relative humidity, $\text{m}^2 (\text{g SO}_4^{2-})^{-1}$

$f(\text{RH})$ accounts for the relative increase in scattering due to relative humidity

Q_{SO_2} is the source strength of anthropogenic SO_2 g S yr^{-1}

$Y_{\text{SO}_4^{2-}}$ is the fractional yield of emitted SO_2 that reacts to produce sulfate aerosol

MW is the molecular weight

$\tau_{\text{SO}_4^{2-}}$ is the sulfate lifetime in the atmosphere, yr

A is the area of the geographical region under consideration, m^2

Charlson, Schwartz, Hales, Cess, Coakley, Hansen & Hofmann, Science, 1992

EVALUATION OF GLOBAL MEAN DIRECT RADIATIVE FORCING DUE TO ANTHROPOGENIC SULFATE

	Quantity	Central Value	Units	Uncertainty Factor
	F_{T}	1370	W m ⁻²	—
	1-A _c	0.4	—	1.1
	T	0.76	—	1.15
	1-R _s	0.85	—	1.1
	$\bar{\beta}$	0.29	—	1.3
<div>$\alpha^* = 8.5$ m² (g SO₄²⁻)⁻¹</div>	$\alpha_{\text{SO}_4^{2-}}$	5	m ² (g SO ₄ ²⁻) ⁻¹	1.5
	$f(\text{RH})$	1.7	—	1.2
<div>Column Burden 4 mg SO₄²⁻ m⁻²</div>	Q_{SO_2}	80	Tg S yr ⁻¹	1.15
	$Y_{\text{SO}_4^{2-}}$	0.4	—	1.5
	$\tau_{\text{SO}_4^{2-}}$	0.02	yr	1.5
	A	5 × 10 ¹⁴	m ²	—
<div>Optical Depth = 0.03</div>	$\overline{\Delta F_{\text{R}}}$	-1.1	W m ⁻²	2.4

Total uncertainty factor evaluated as $f_t = \exp\left[\sum (\log f_i)^2\right]^{1/2}$

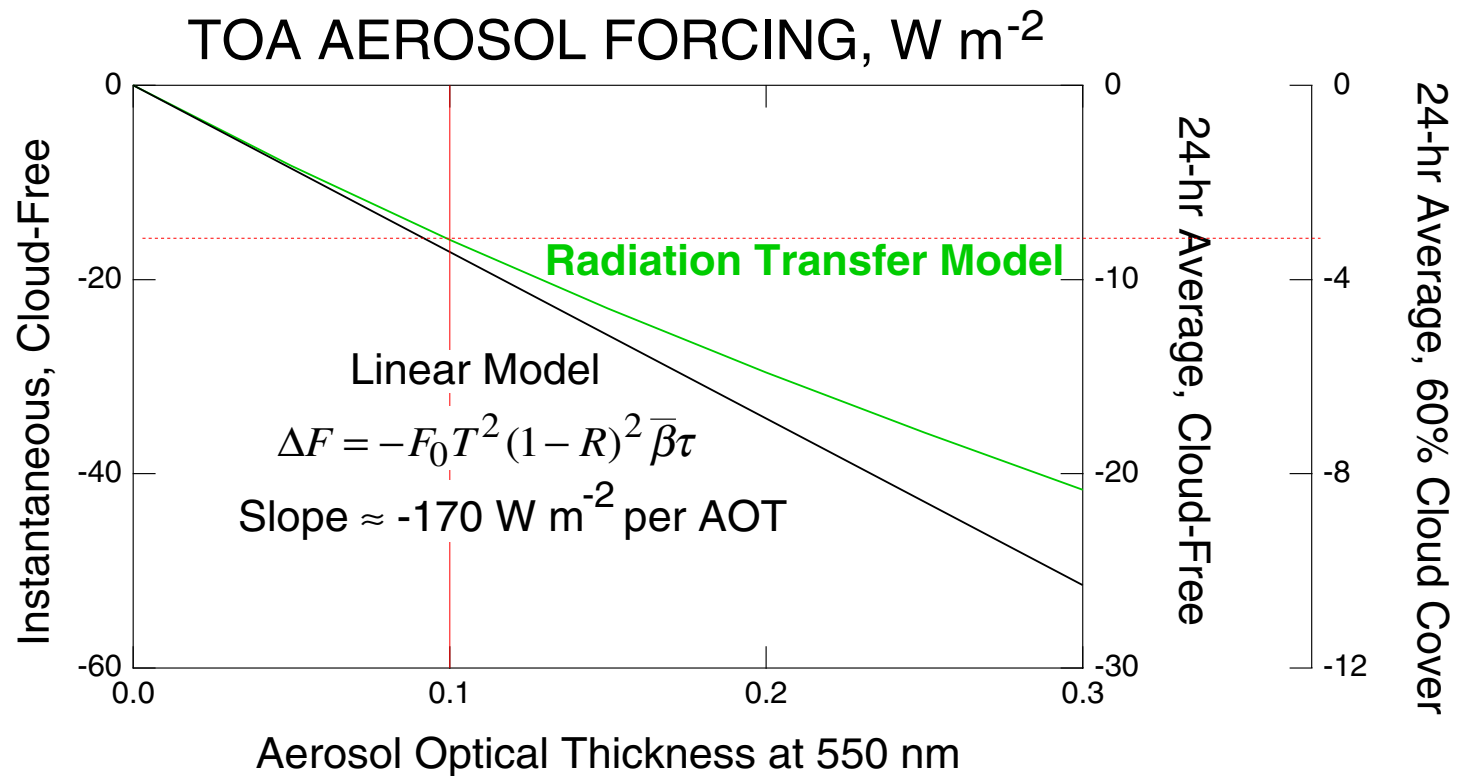
Penner, Charlson, Hales, Laulainen, Leifer, Novakov,
Ogren, Radke, Schwartz & Travis, BAMS, 1994

DIRECT AEROSOL FORCING AT TOP OF ATMOSPHERE

Dependence on Aerosol Optical Thickness

Comparison of Linear Formula and Radiation Transfer Model

Particle radius $r = 85$ nm; surface reflectance $R = 0.15$; single scatter albedo $\omega_0 = 1$.

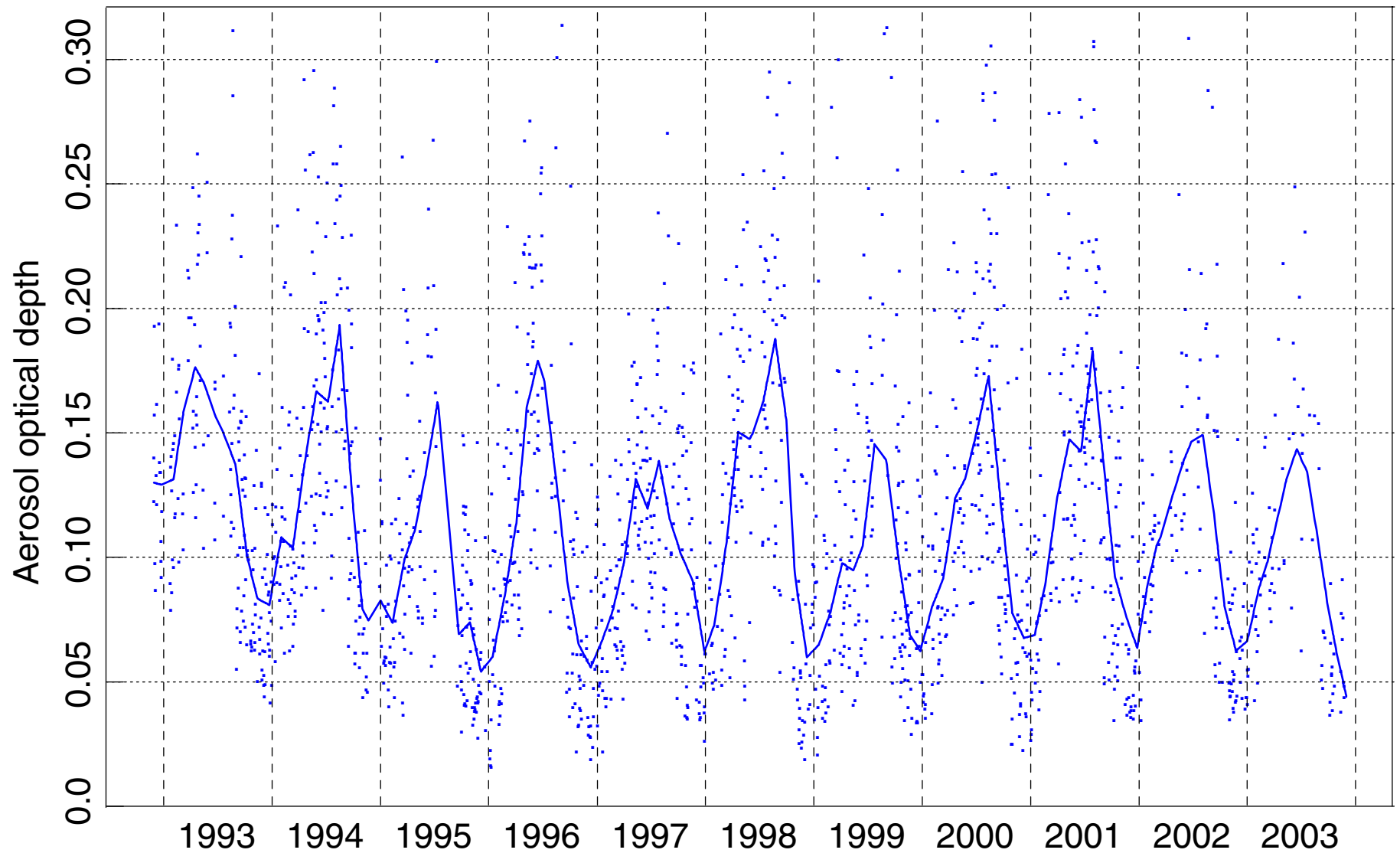


Global-average AOT 0.1 corresponds to global-average forcing -3.2 W m^{-2} .

AEROSOL OPTICAL DEPTH

Determined by sunphotometry

North central Oklahoma - Daily average at 500 nm



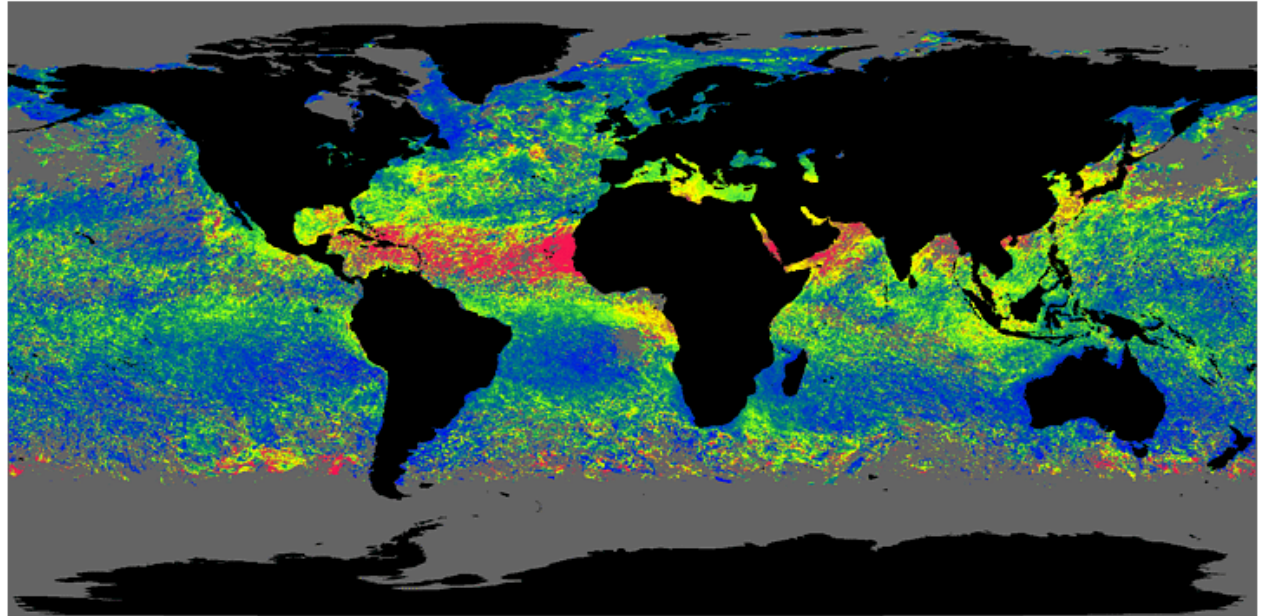
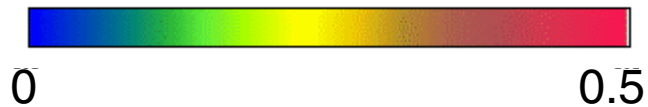
J. Michalsky et al., JGR, 2001

MONTHLY AVERAGE AEROSOL JUNE 1997

Polder radiometer on Adeos satellite

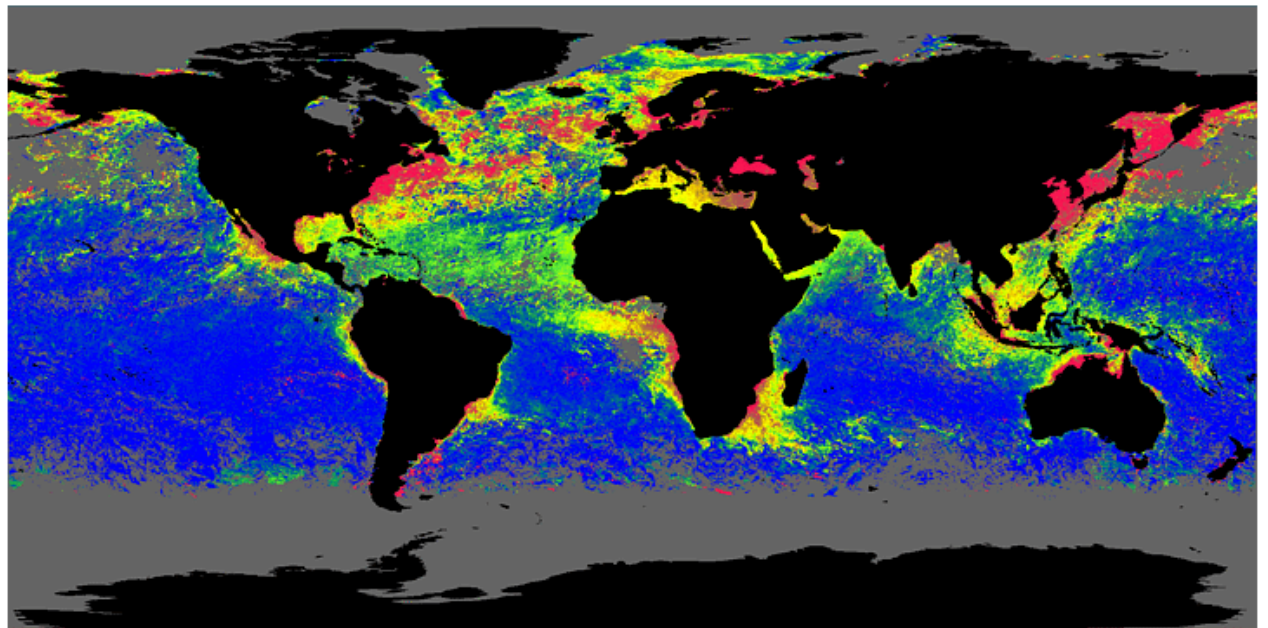
Optical Thickness τ

$\lambda = 865 \text{ nm}$



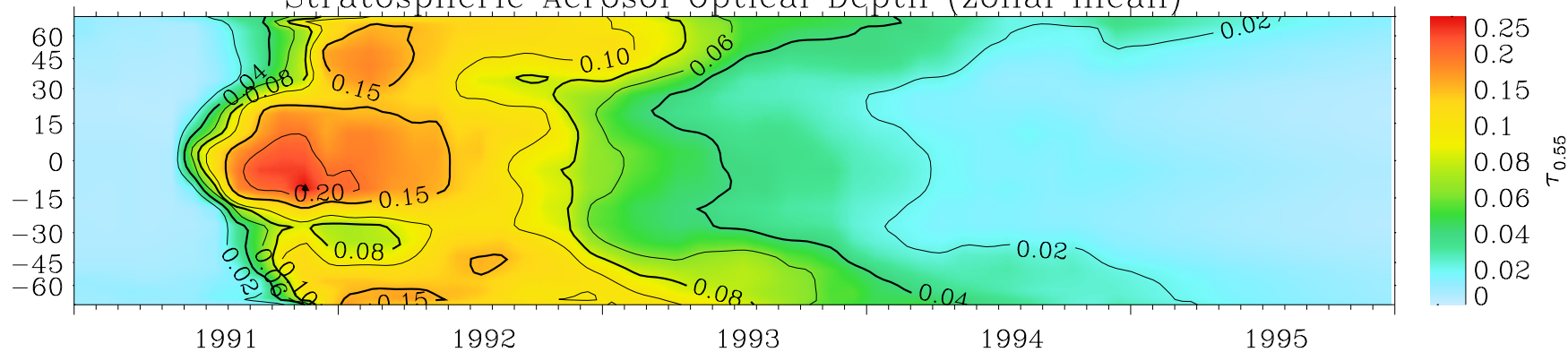
Ångström Exponent α

$$\alpha = -d \ln \tau / d \ln \lambda$$

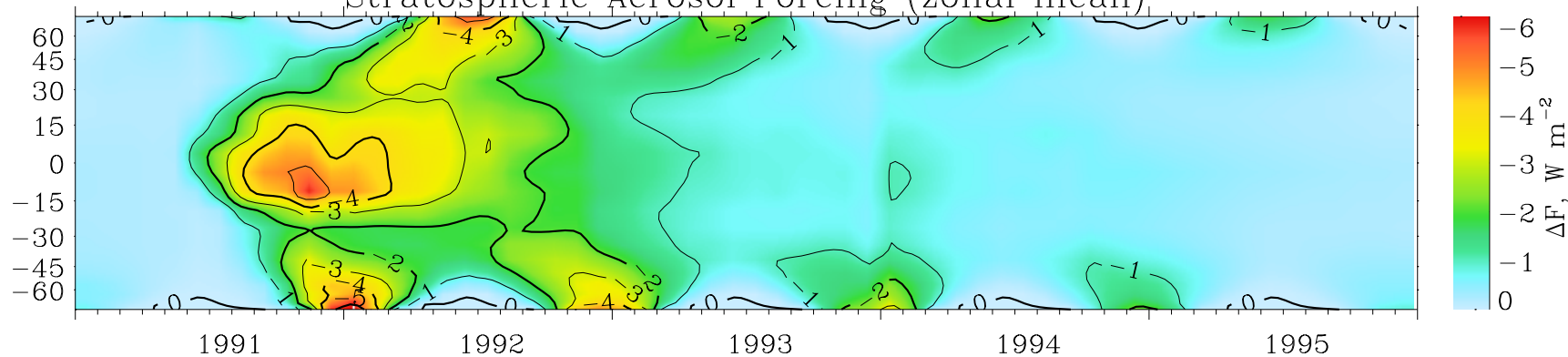


Influence of Pinatubo Eruption on Aerosol Forcing and Global Temperature

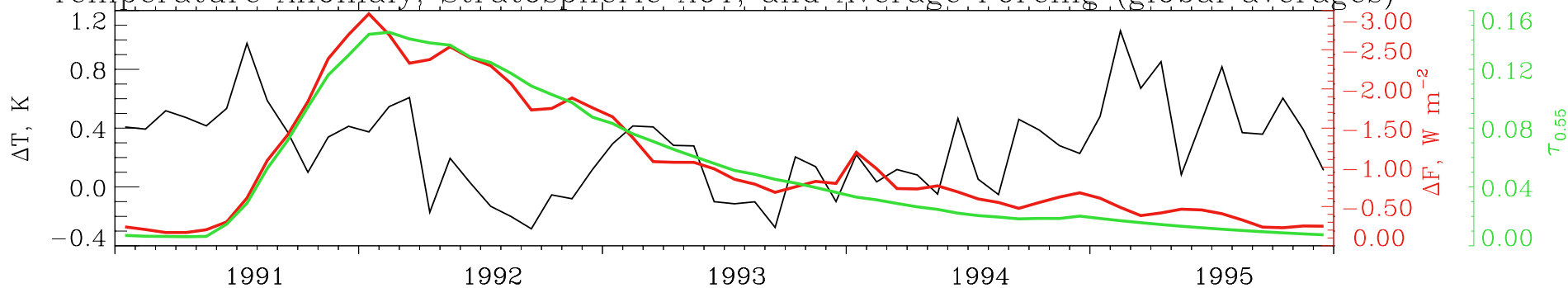
Stratospheric Aerosol Optical Depth (zonal mean)



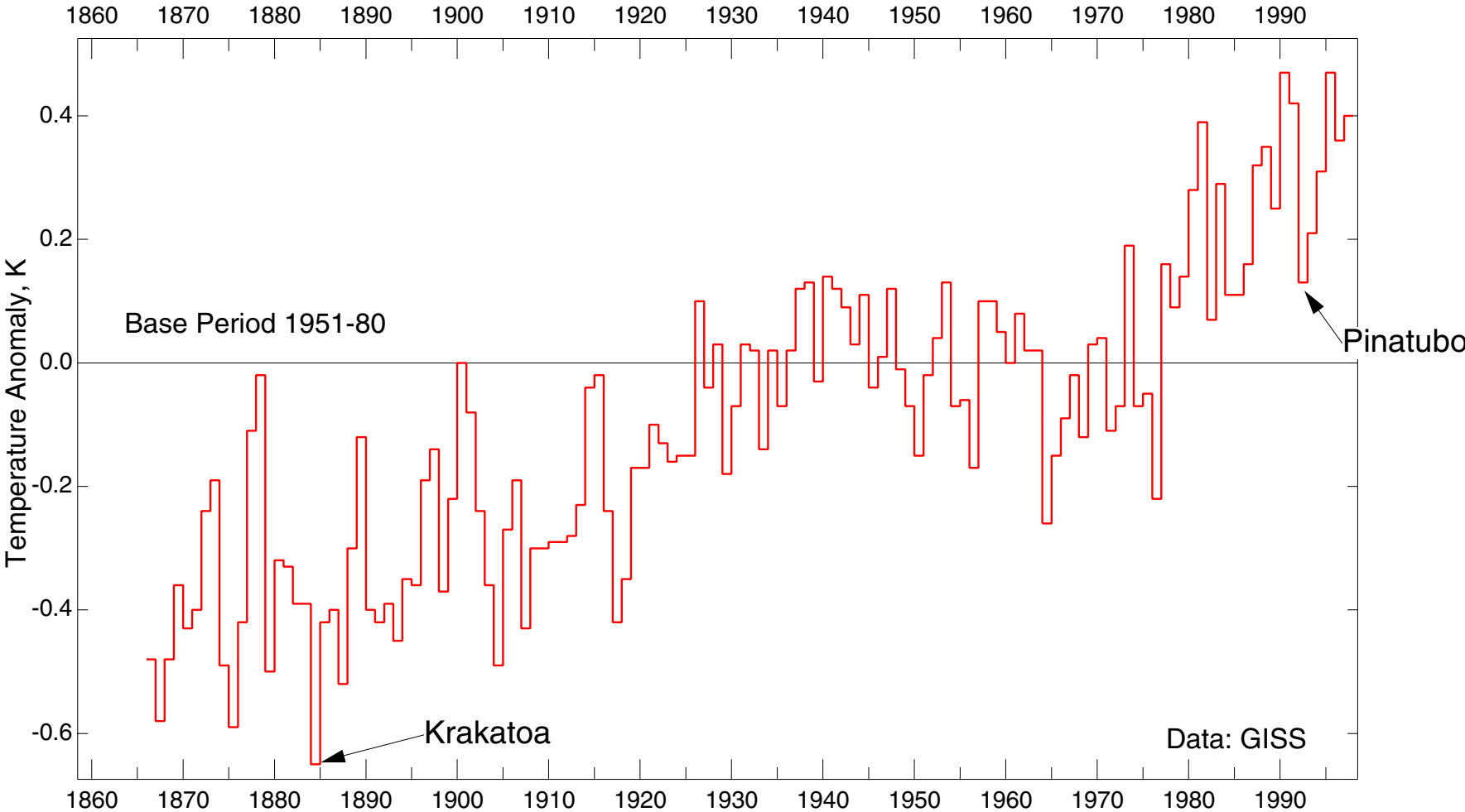
Stratospheric Aerosol Forcing (zonal mean)



Temperature Anomaly, Stratospheric AOT, and Average Forcing (global averages)



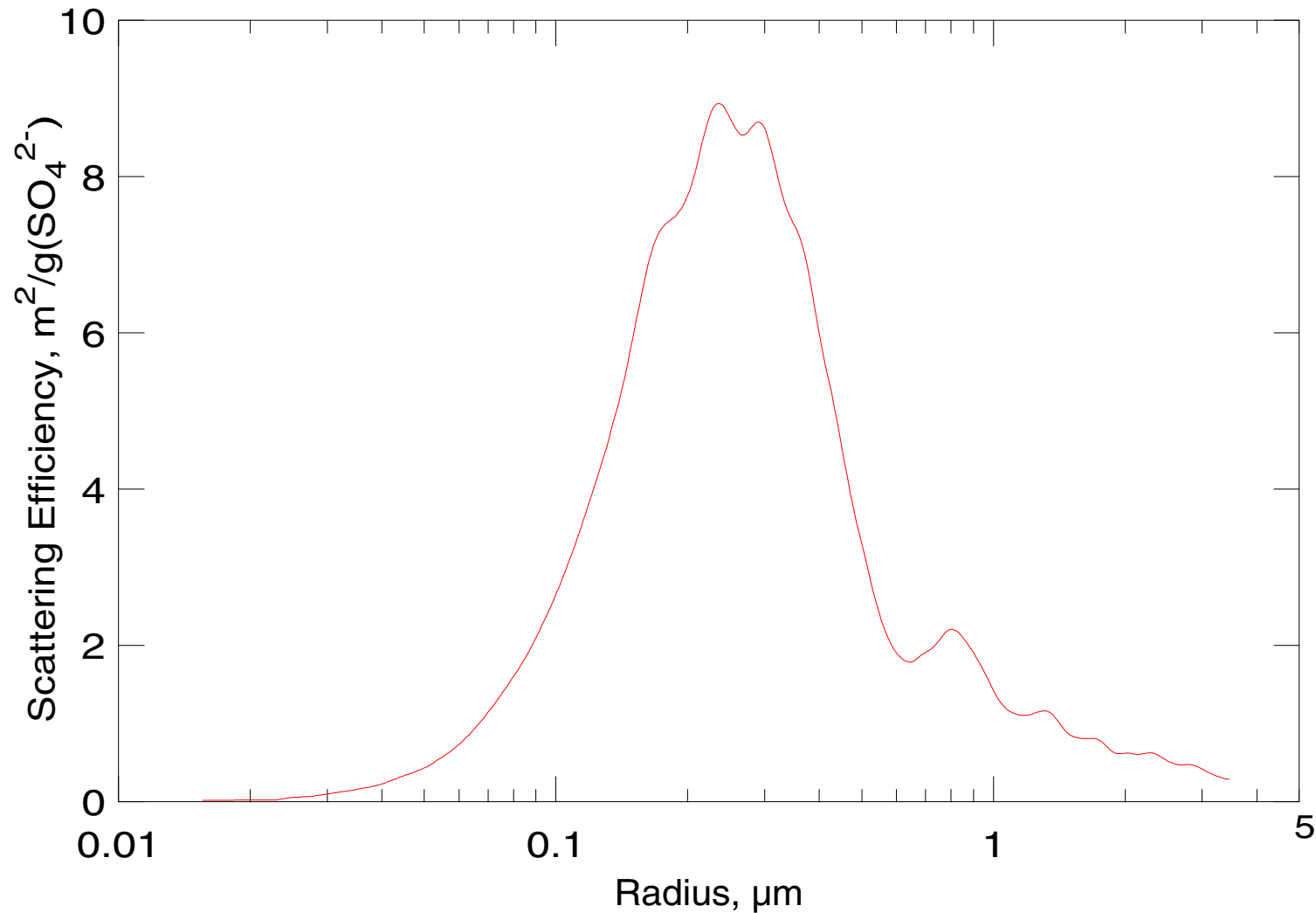
GLOBAL TEMPERATURE TREND OVER THE INDUSTRIAL PERIOD



LIGHT SCATTERING EFFICIENCY

Dependence on particle radius -- Size matters!

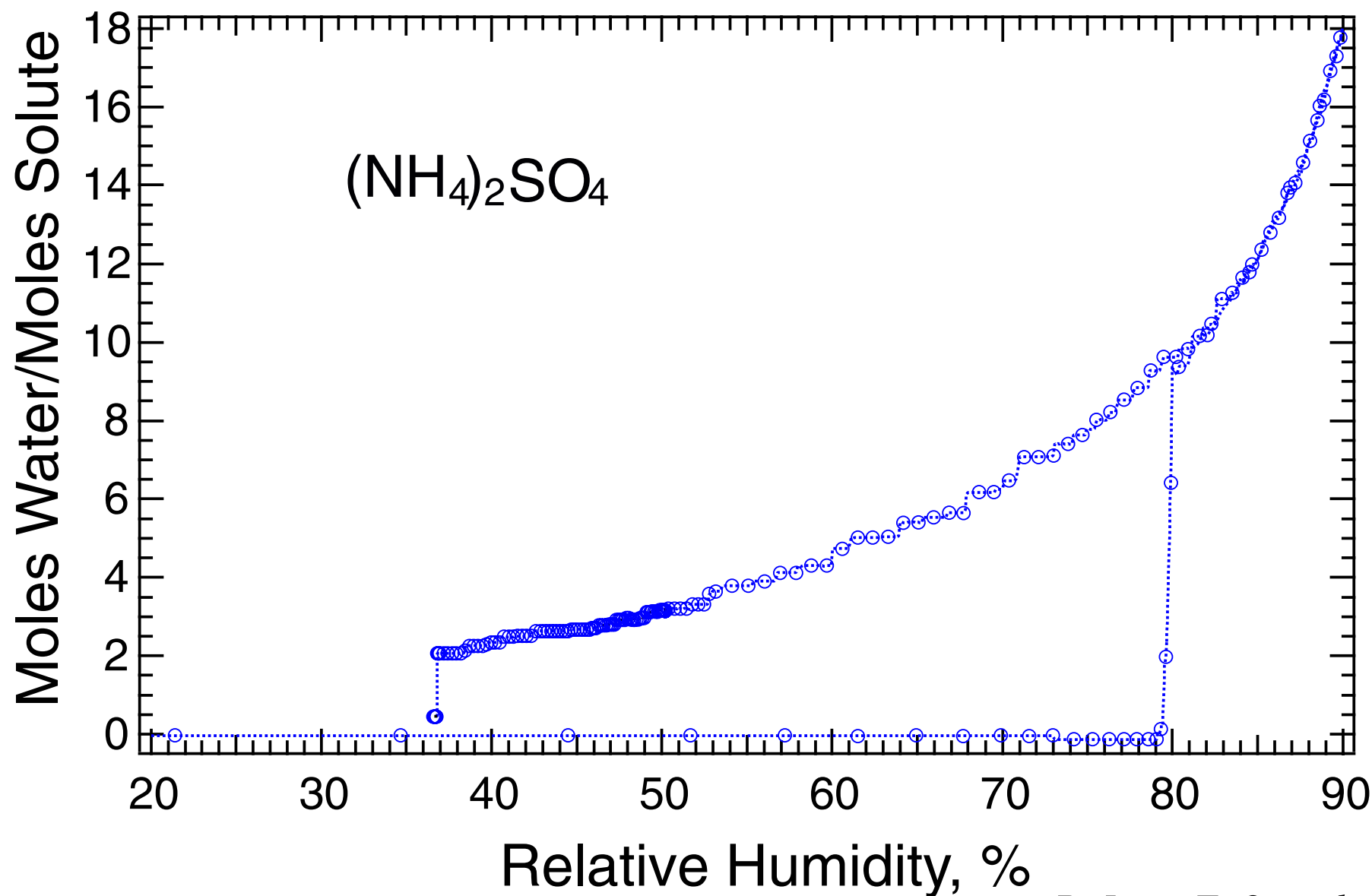
Ammonium Sulfate, 530 nm



Data of Ouimette and Flagan, 1982

WATER UPTAKE BY HYGROSCOPIC PARTICLE

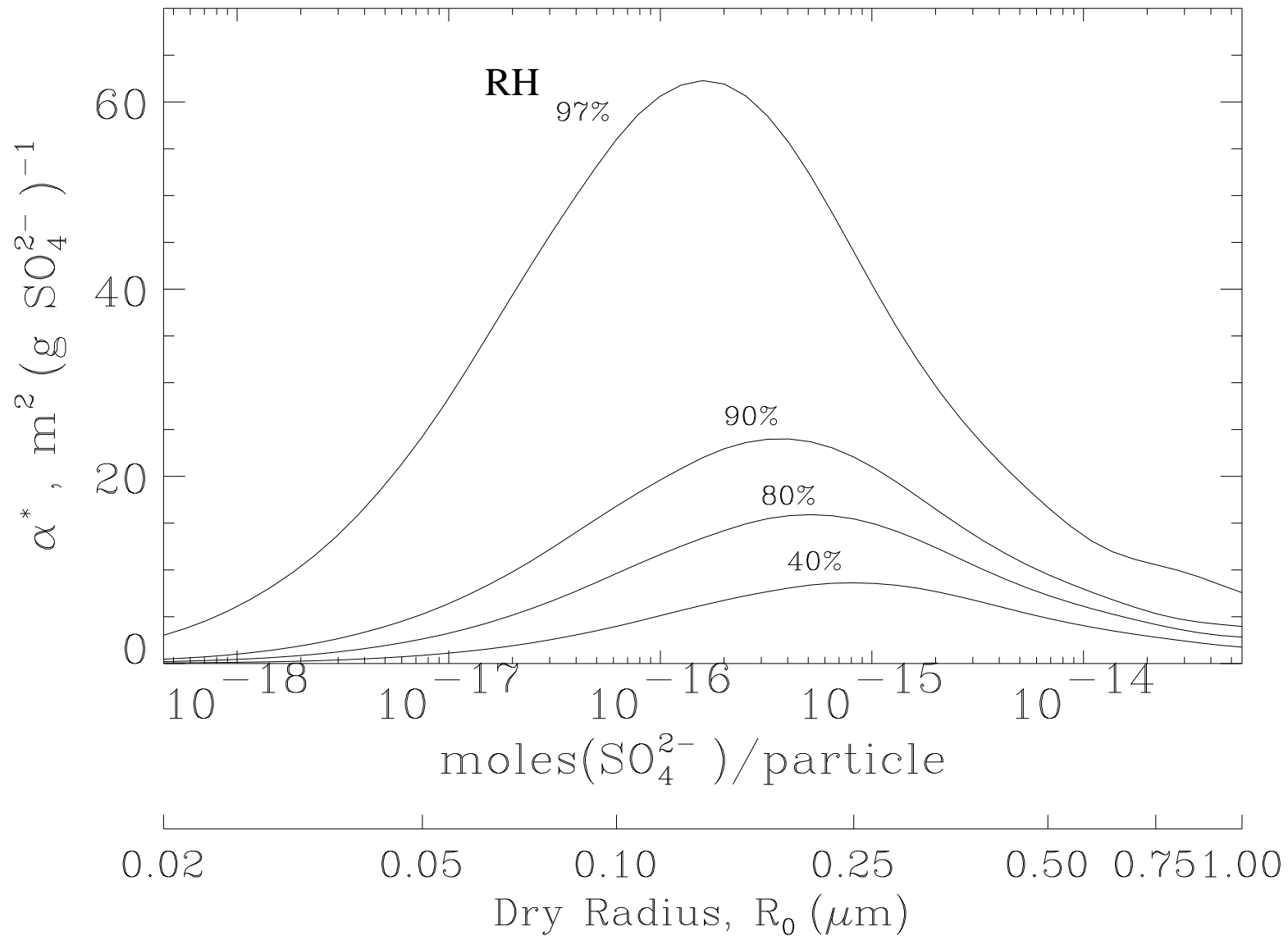
Dependence on relative humidity



D. Imre, T. Onasch, BNL

LIGHT SCATTERING EFFICIENCY OF $(\text{NH}_4)_2\text{SO}_4$

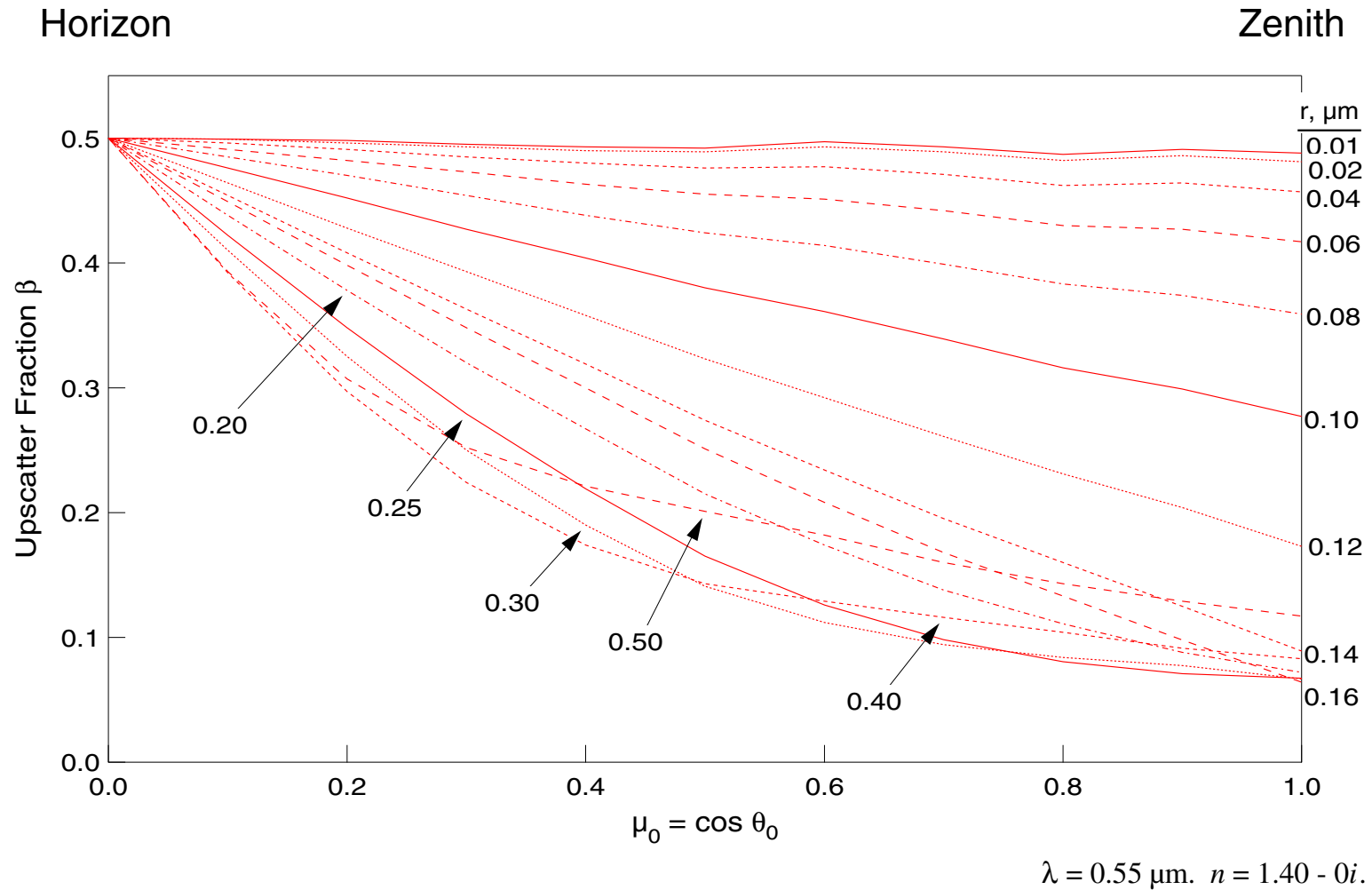
DEPENDENCE ON PARTICLE SIZE AND RH



Nemesure et al., JGR, 1995

UPSCATTER FRACTION

Dependence on solar zenith angle and particle radius



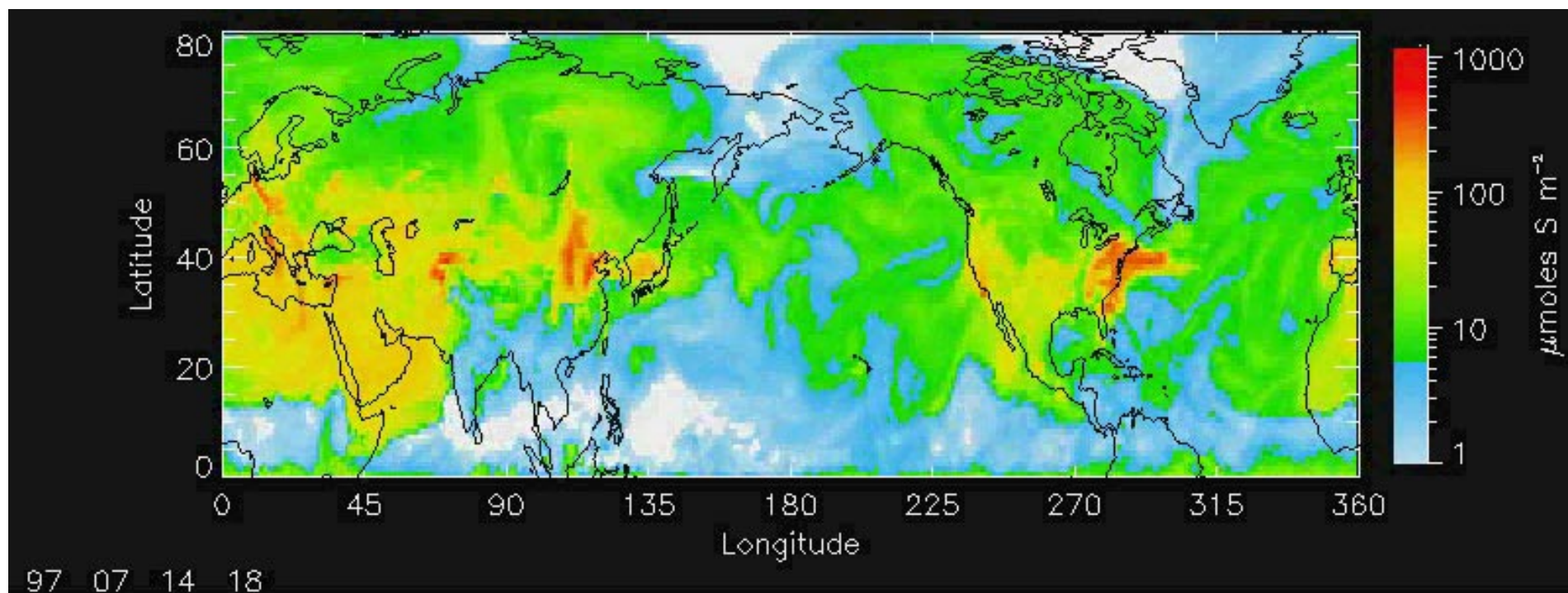
For sun at horizon $\beta = 0.5$ (by symmetry).

For small particles, $r \ll \lambda$, upscatter fraction approaches that for Rayleigh scattering (0.5).

HEMISPHERIC DISTRIBUTION OF SULFATE COLUMN BURDEN

Vertical integral of concentration

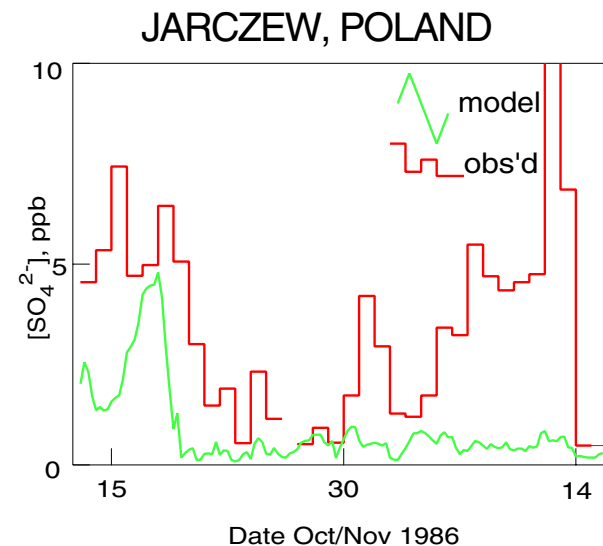
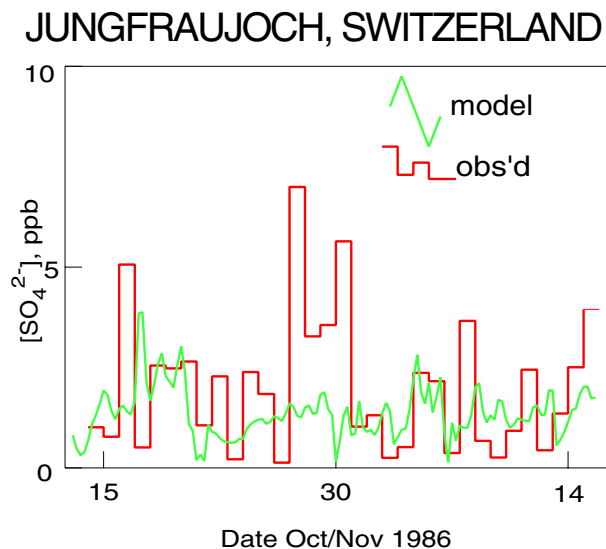
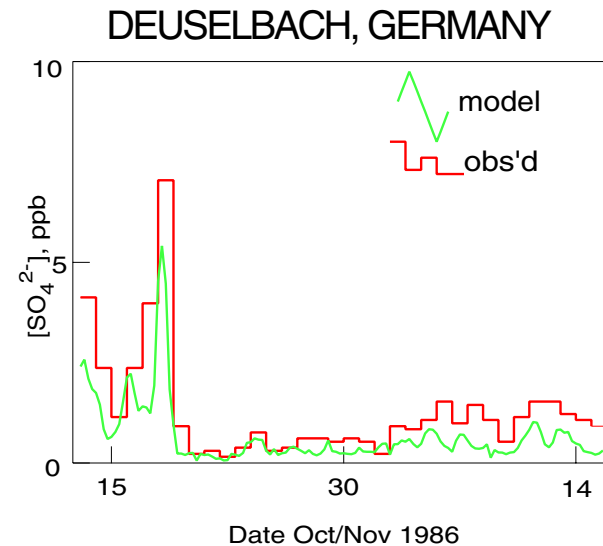
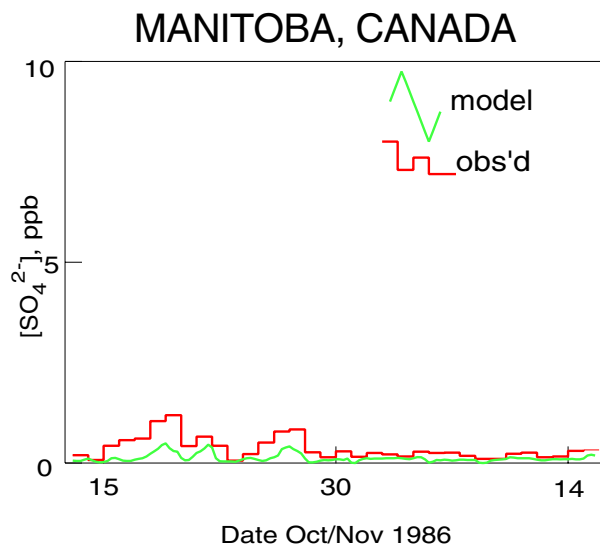
July 14, 1997, 1800 UTC



Brookhaven National Laboratory Chemical Transport Model

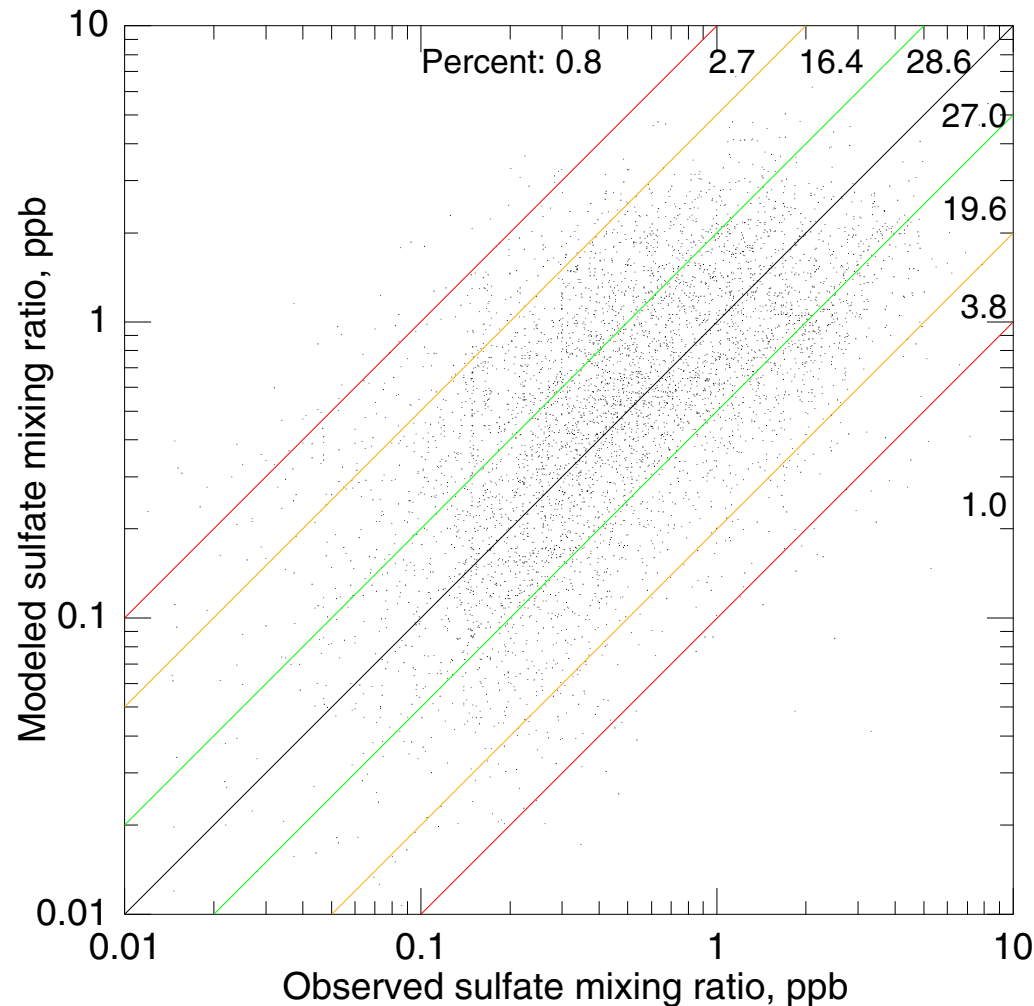
COMPARISON OF MODEL AND OBSERVATIONS

Comparisons for 24-hr sulfate mixing ratio at surface



MODEL-OBSERVATION COMPARISONS

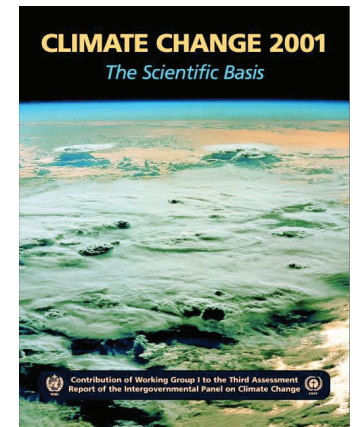
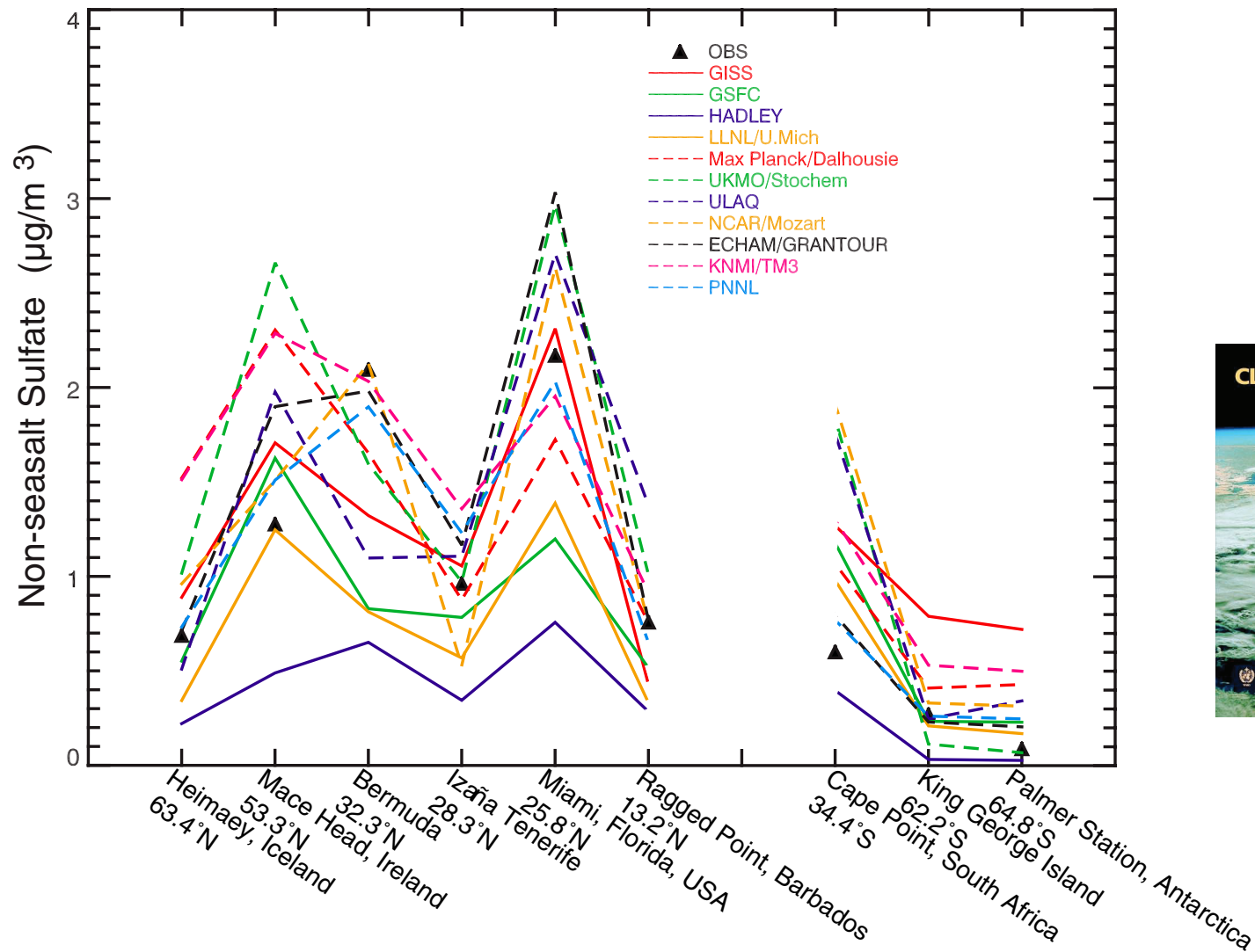
5083 24-Hour sulfate mixing ratio in BNL CTM driven by assimilated meteorological data - June-July 1997



56% of comparisons within factor of 2. 92% within factor of 5.

SULFATE MODEL INTERCOMPARISON

Annual average non-seasalt sulfate in 11 chemical transport models and comparison with observations at nine stations



Penner et al., IPCC, 2001

“Most models predict surface-level seasonal mean sulphate aerosol mixing ratios to within 20%.”

“We cannot be sure that these models achieve reasonable success for the right reasons.”

DO YOU HAVE A FEW MOMENTS?

The Problem

How to represent the size-distribution of atmospheric aerosols and its evolution in chemical transport models

The Solution

Represent the size distribution in terms of its low-order moments

$$\mu_k \equiv \int_0^\infty r^k \left(\frac{dN}{dr} \right) dr$$



APPLICATION TO SULFATE IN EASTERN NORTH AMERICA

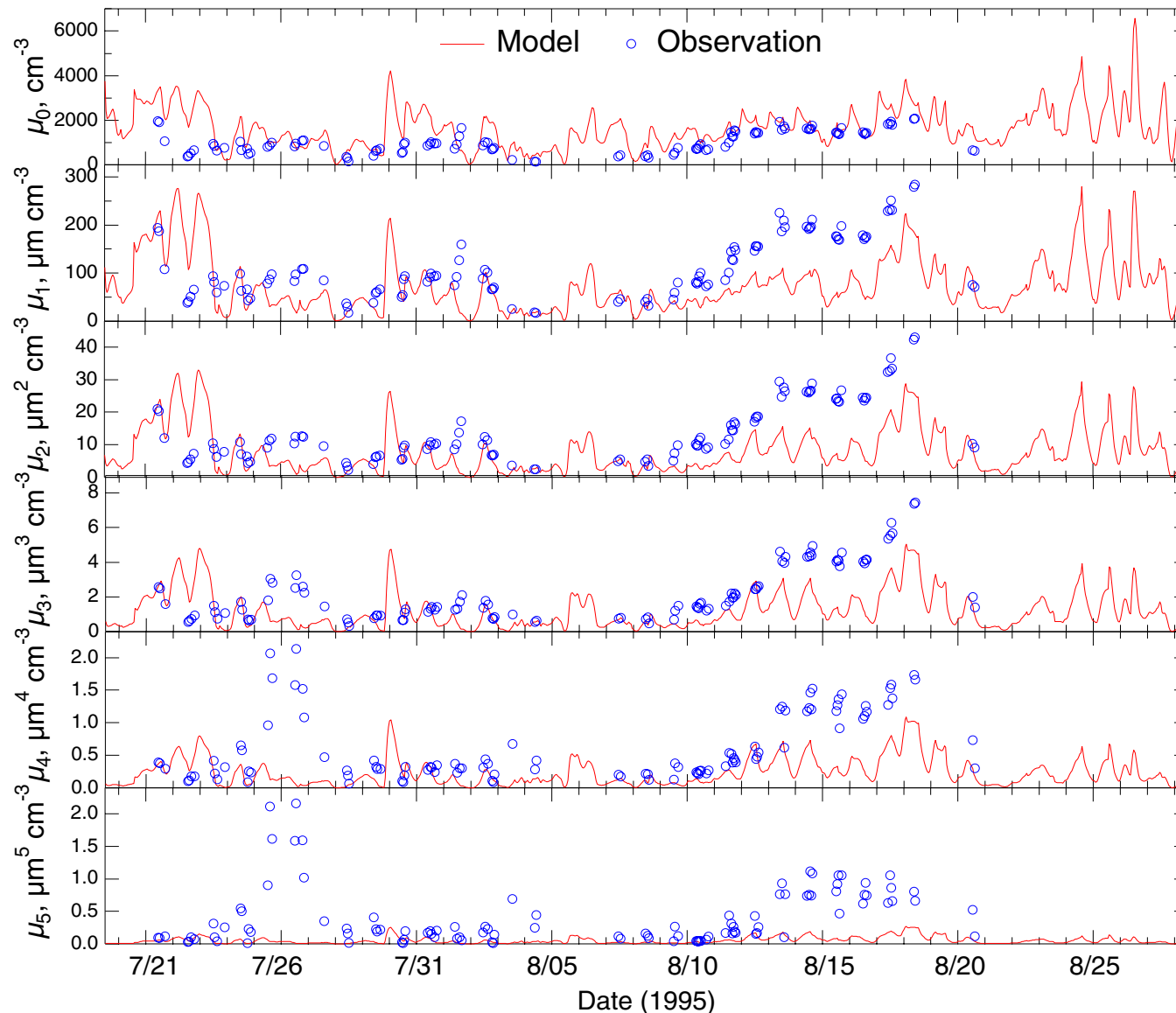
Simulations: 40 days, 19 July to 28 August 1995.

Comparison with observations: Sulfate mass concentration, aerosol number concentration and size distributions at the Great Smoky Mountains National Park during Southeastern Aerosol and Visibility Study.

Limitation: Model is for sulfate only; size measurements are for entire aerosol, not just sulfate.

TIME SERIES COMPARISON FOR AEROSOL MOMENTS

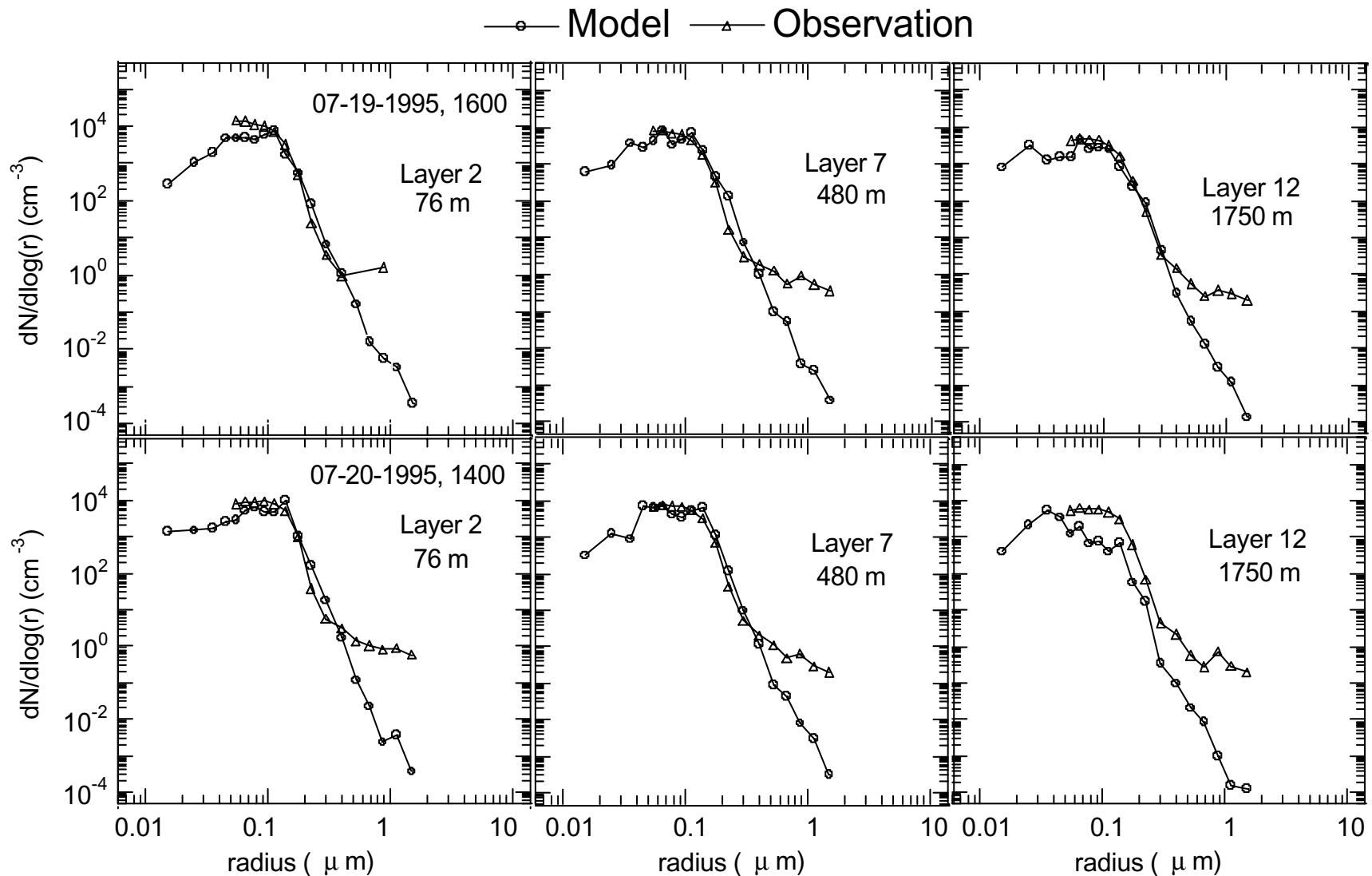
Look Ridge, Great Smoky Mountains TN (84° W, 36° N; 900 m) during SEAVS



Yu, Kasibhatla, Wright, Schwartz, McGraw & Deng, *JGR*, 2003

SIZE DISTRIBUTIONS

Comparison of Measurement and Retrieval from Model
At 3 Altitudes near Nashville TN

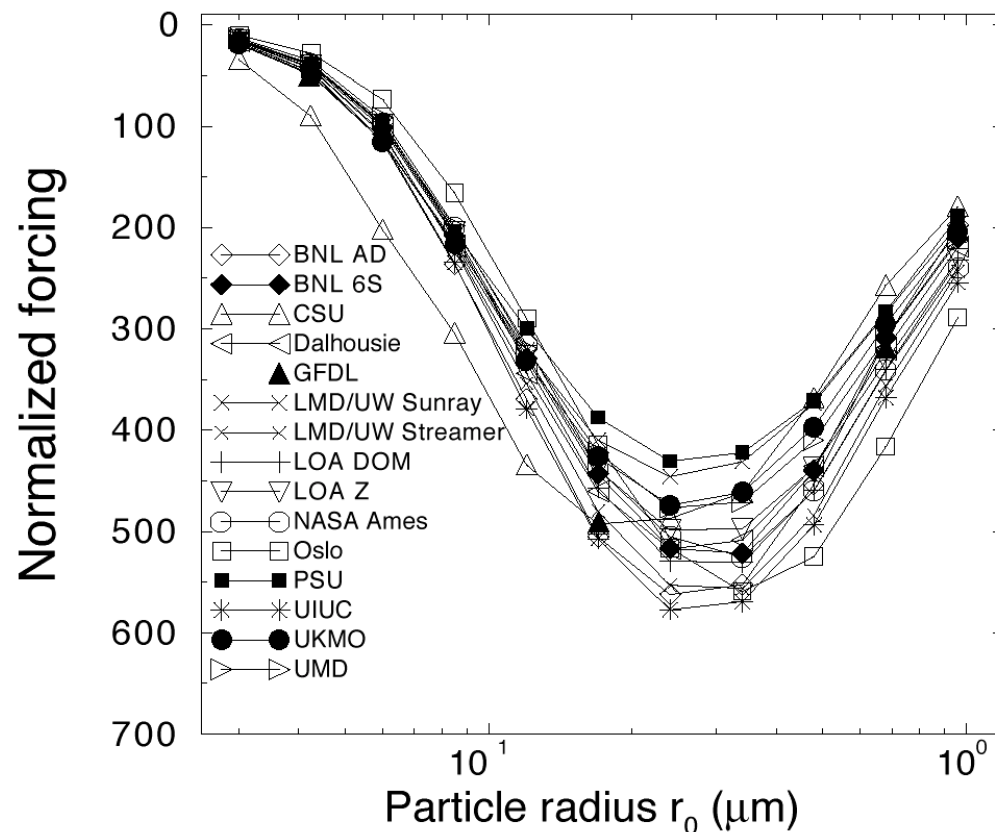


Yu, Kasibhatla, Wright, Schwartz, McGraw & Deng, *JGR*, 2003

INTERCOMPARISON OF BROADBAND SHORTWAVE FORCING BY AMMONIUM SULFATE AEROSOL

Normalized global-average forcing: $\text{W m}^{-2} / \text{g}(\text{SO}_4^{2-}) \text{ m}^{-2}$ or $\text{W} / \text{g}(\text{SO}_4^{2-})$

Aerosol optical depth 0.2; surface albedo 0.15

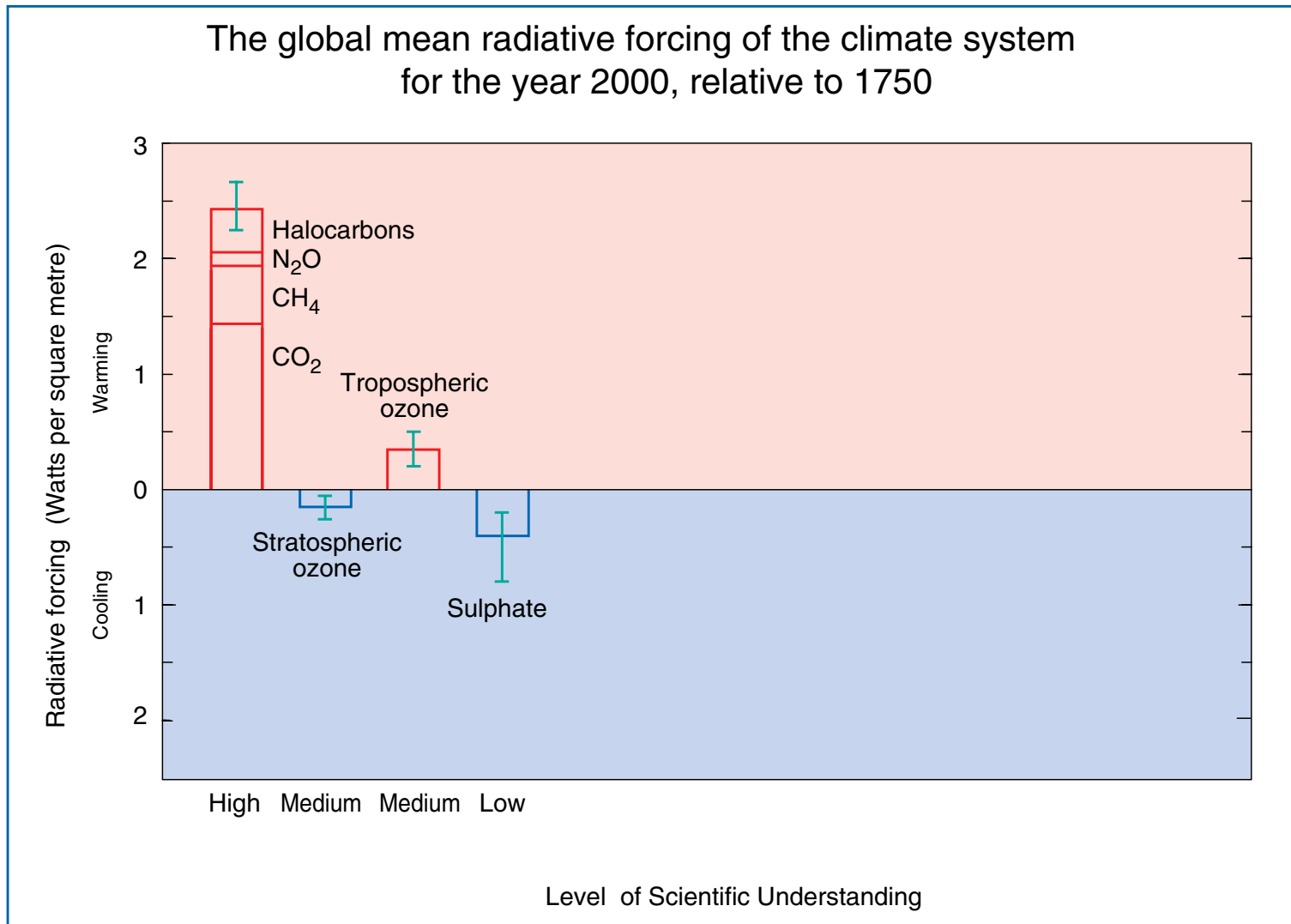


Standard deviation $\sim 8\%$ for 15 models at radius $\sim 200 \text{ nm}$.

Boucher, Schwartz and 28 co-authors, JGR, 1998

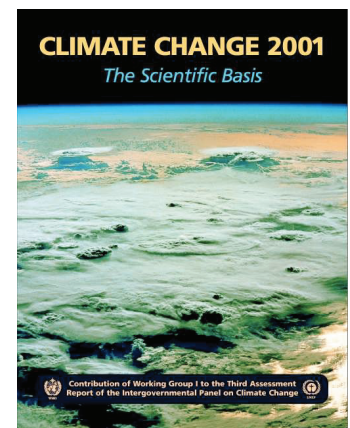
RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD IPCC (2001)

GHG's and sulfate aerosol direct effects



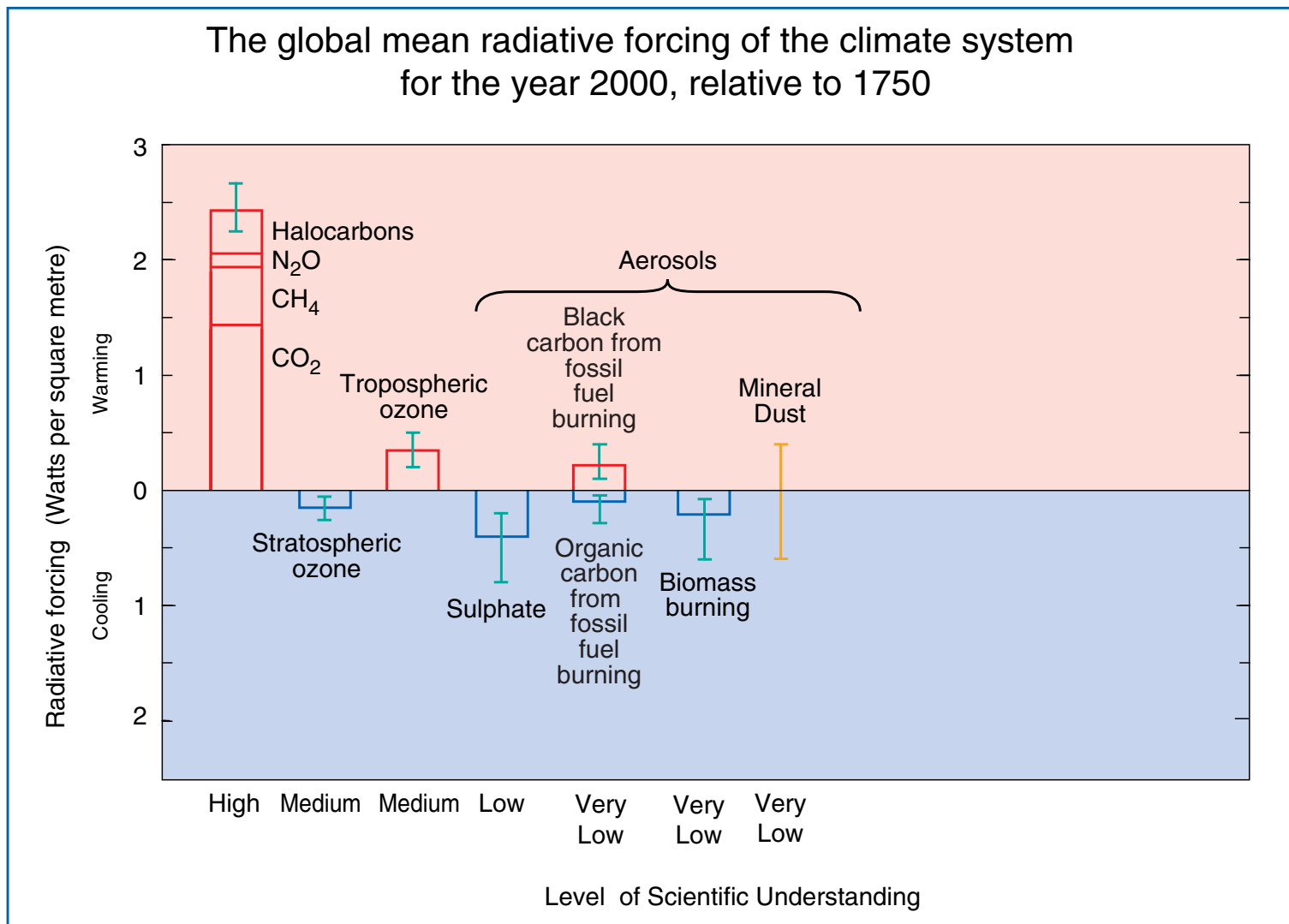
Summary for Policymakers

A Report of Working Group I of the
Intergovernmental Panel on Climate Change

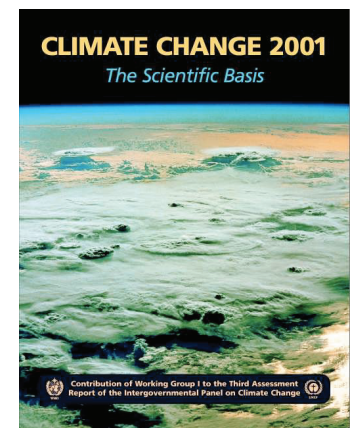


RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD IPCC (2001)

GHG's and aerosol direct effects



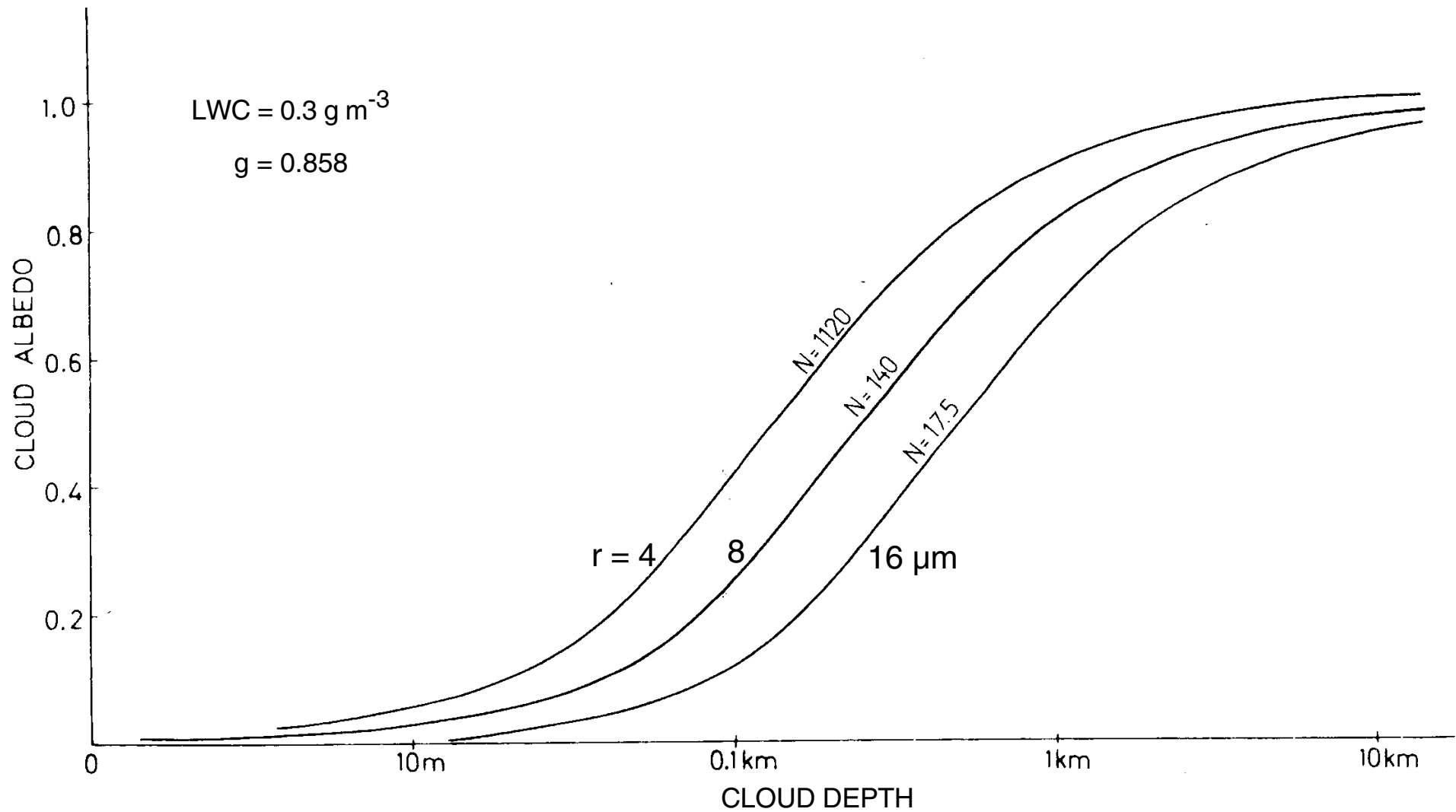
Summary for Policymakers A Report of Working Group I of the Intergovernmental Panel on Climate Change



INDIRECT EFFECT

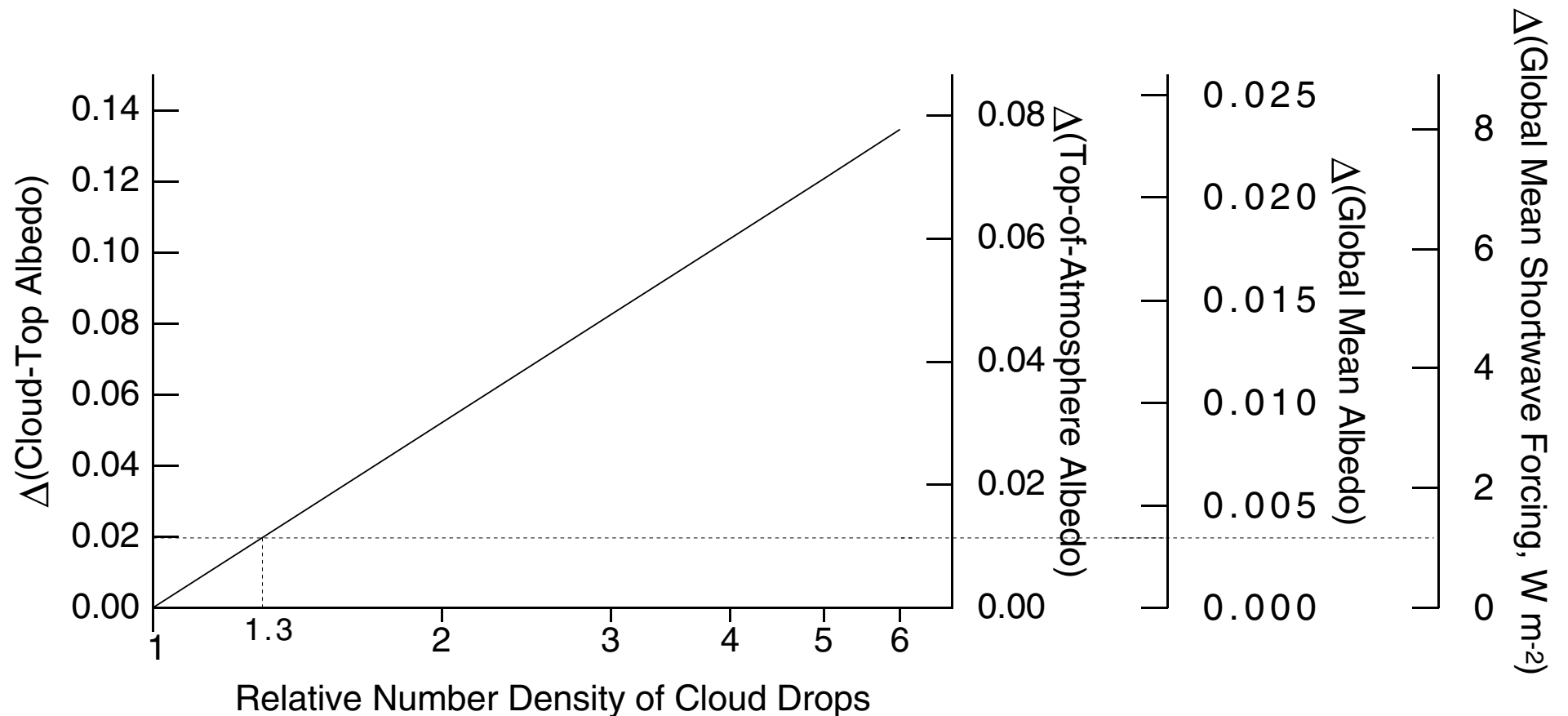
DEPENDENCE OF CLOUD ALBEDO ON CLOUD DEPTH

Influence of Cloud Drop Radius and Concentration



Twomey, *Atmospheric Aerosols*, 1977

SENSITIVITY OF ALBEDO AND FORCING TO CLOUD DROP CONCENTRATION



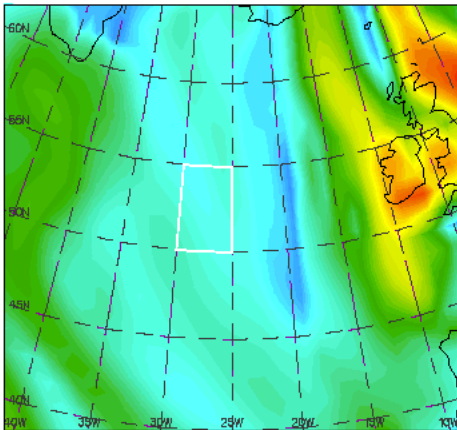
Schwartz and Slingo (1996)

MODELED SULFATE COLUMN BURDEN

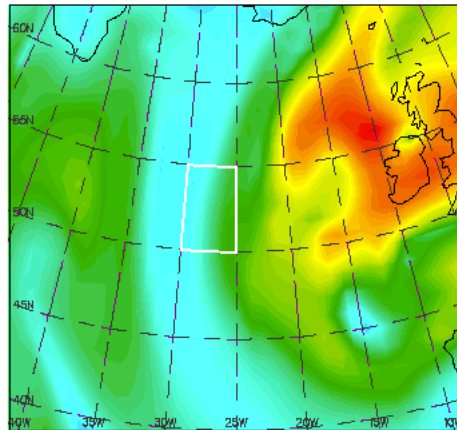
$$\int [\text{SO}_4^{2-}] dz$$

April 2-8, 1987

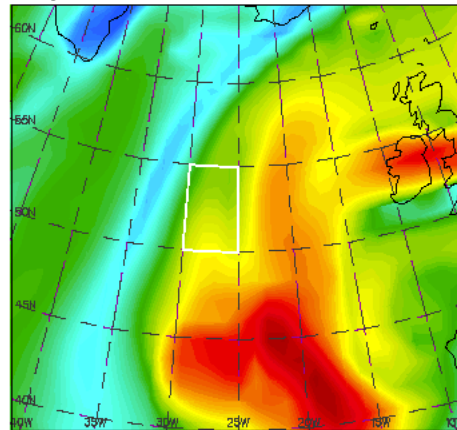
April 2



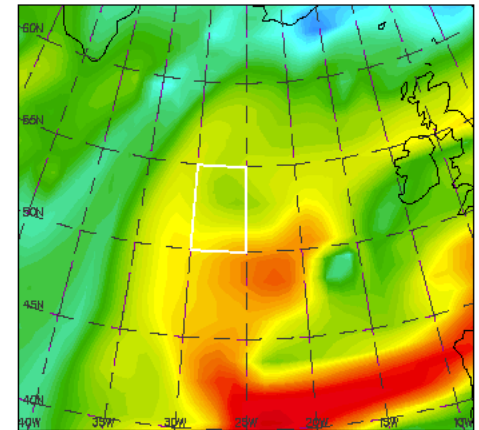
April 3



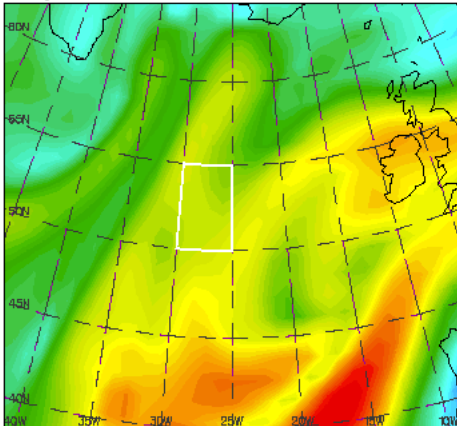
April 4



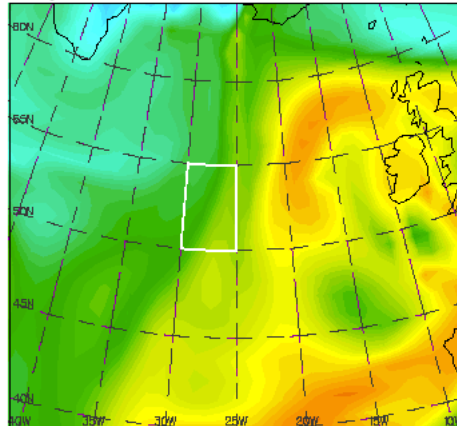
April 5



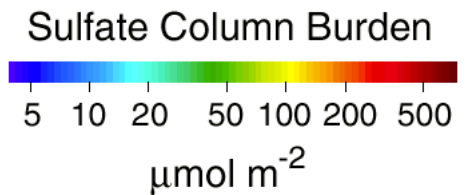
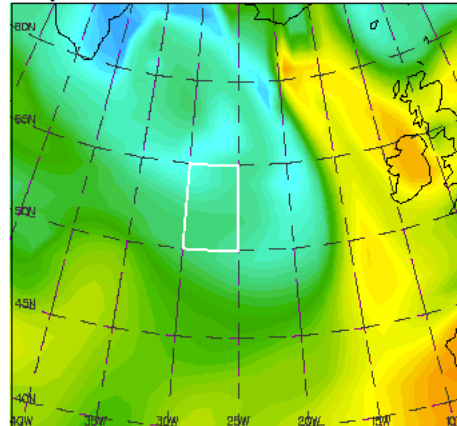
April 6



April 7



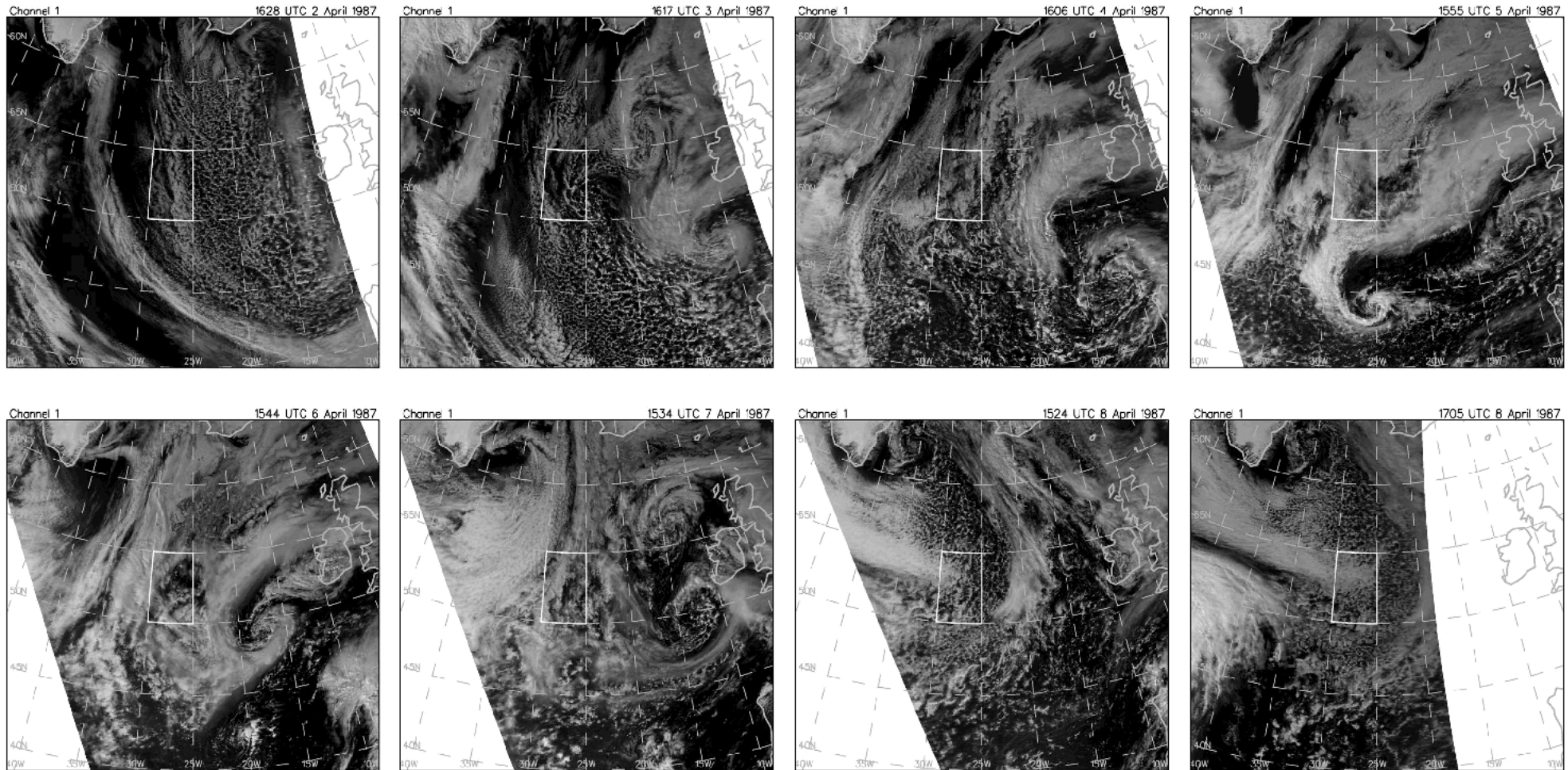
April 8



Schwartz, Harshvardhan & Benkovitz, PNAS, 2002

AVHRR IMAGES APRIL 2-8, 1987

Channel 1, Visible, 0.58-0.68 μm

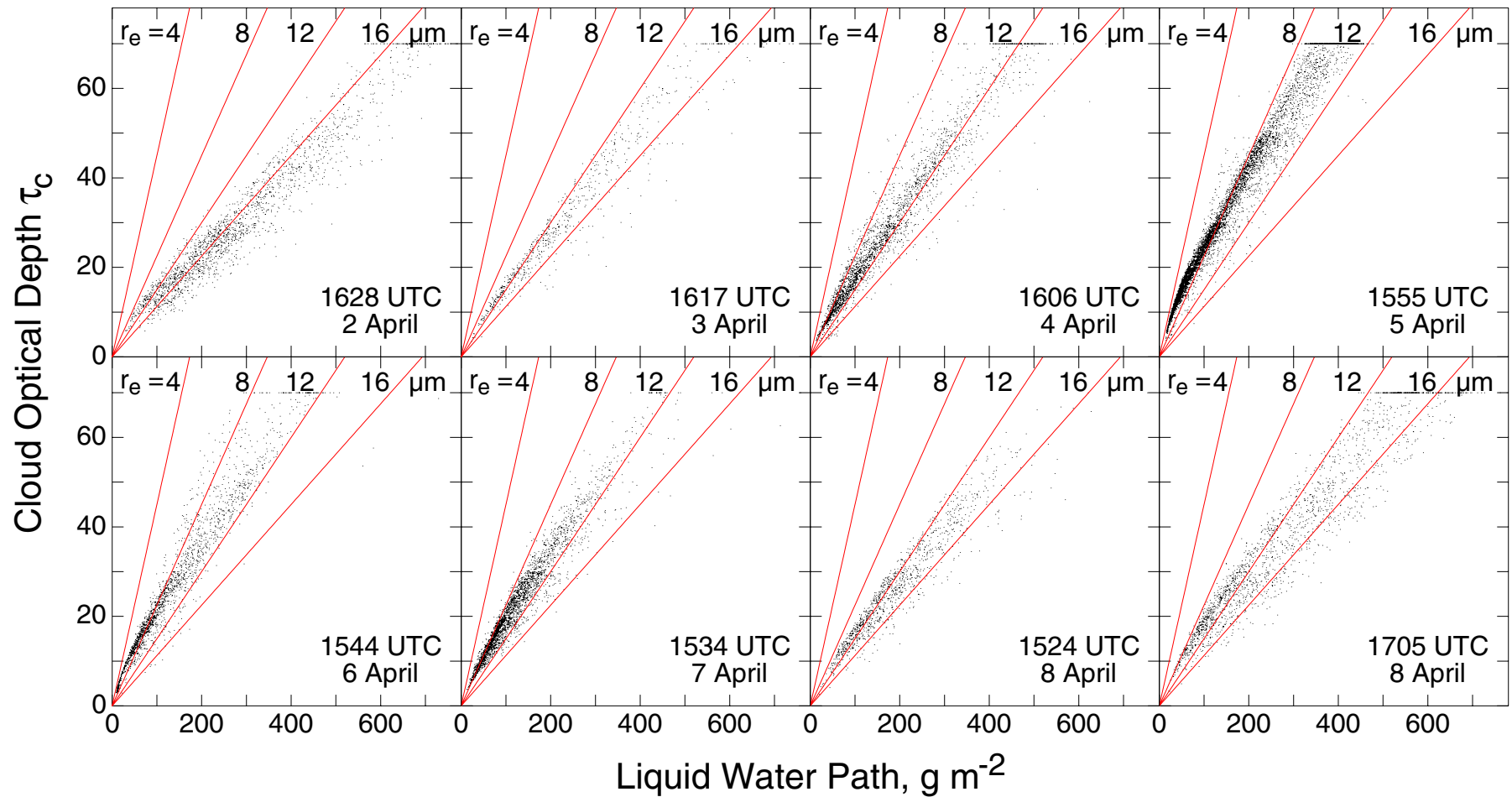


Harshvardhan, Schwartz, Benkovitz and Guo, J Atmos Sci, 2002

CLOUD OPTICAL DEPTH

Dependence on Liquid Water Path

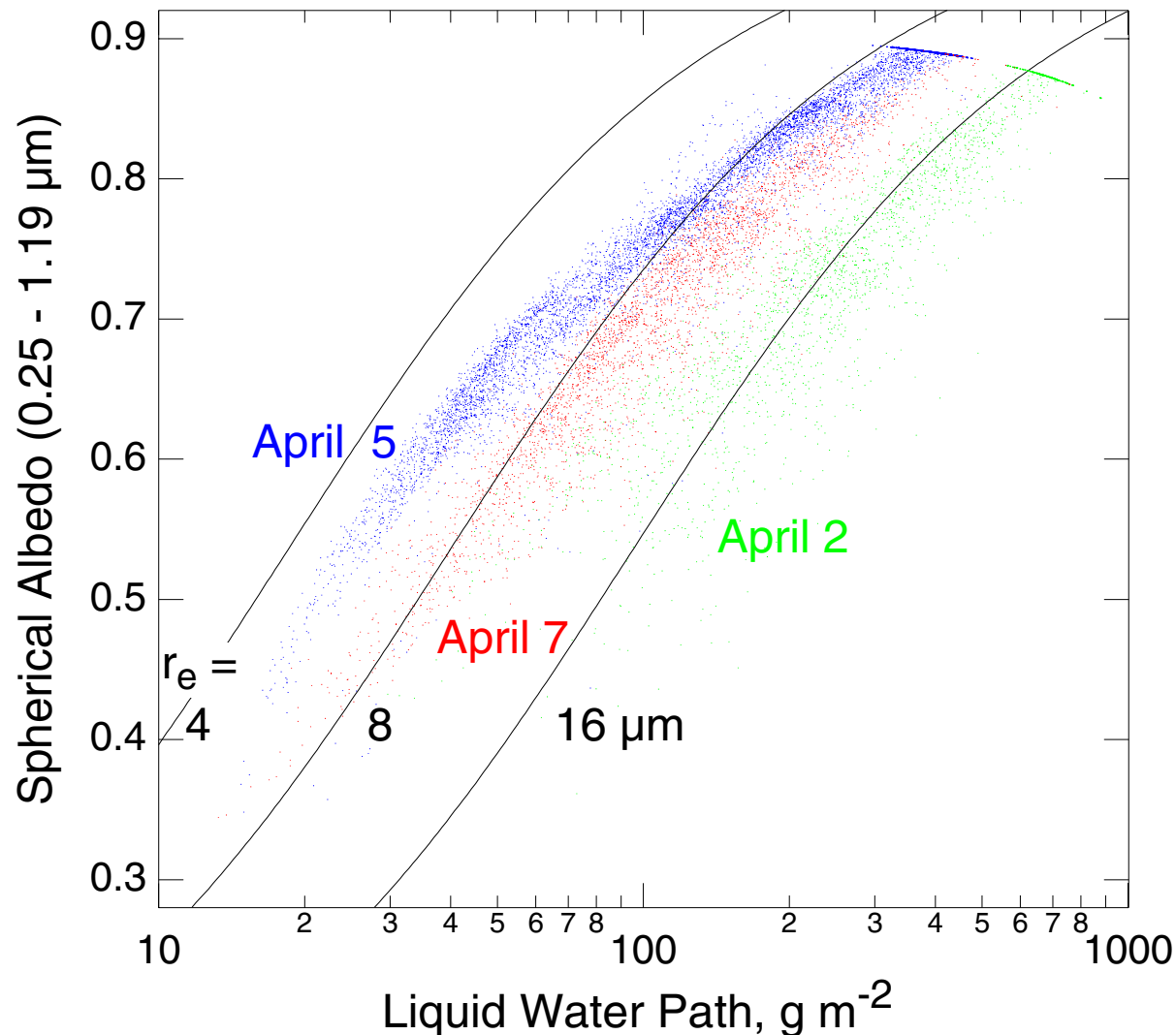
25°-30°W, 50°-55°N April 2-8, 1987



CLOUD-TOP ALBEDO

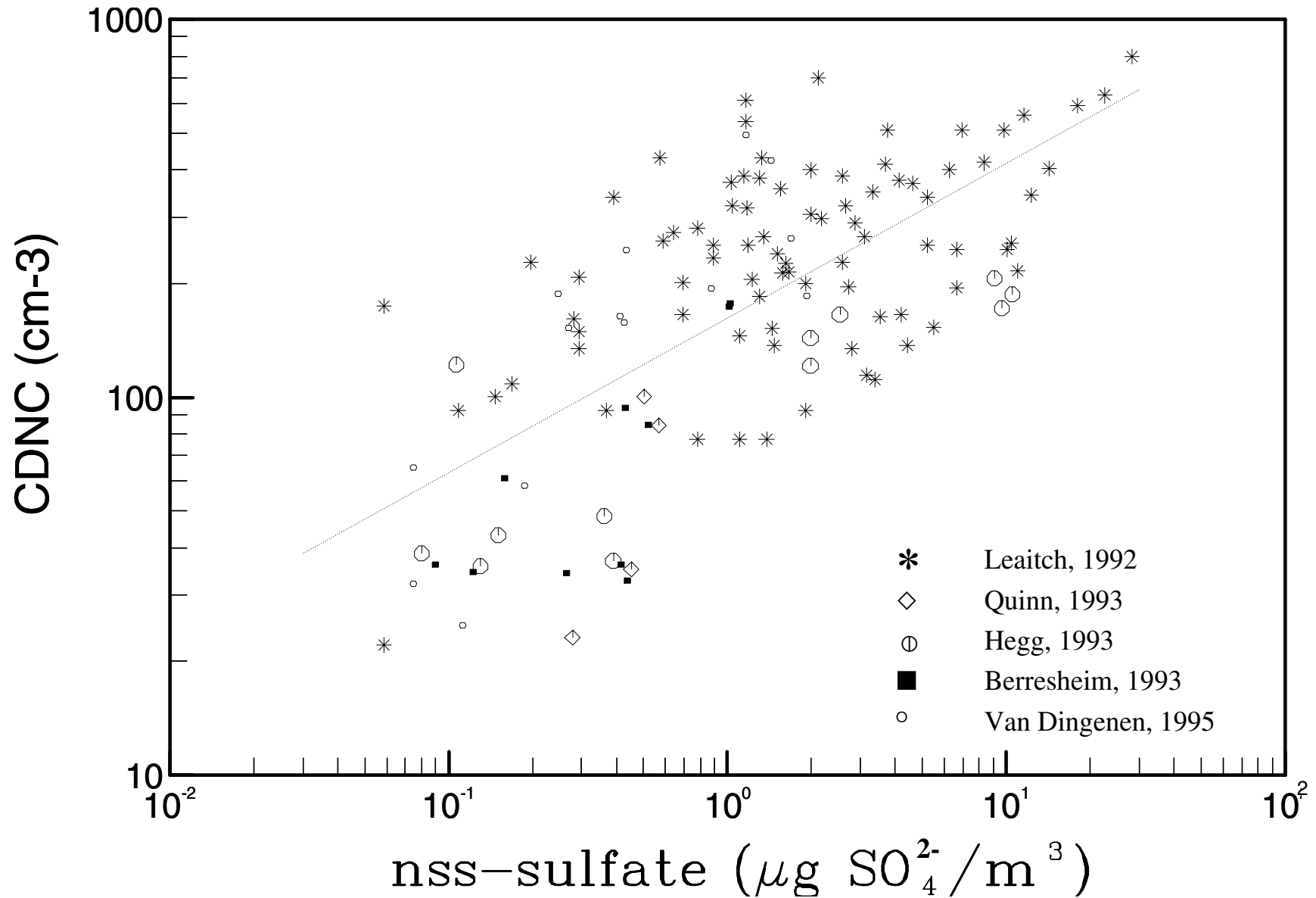
Dependence on Liquid Water Path

25°-30°W, 50°-55°N April 2, 5 and 7, 1987



CLOUD DROPLET NUMBER CONCENTRATION

Dependence on Non-Seasalt Sulfate

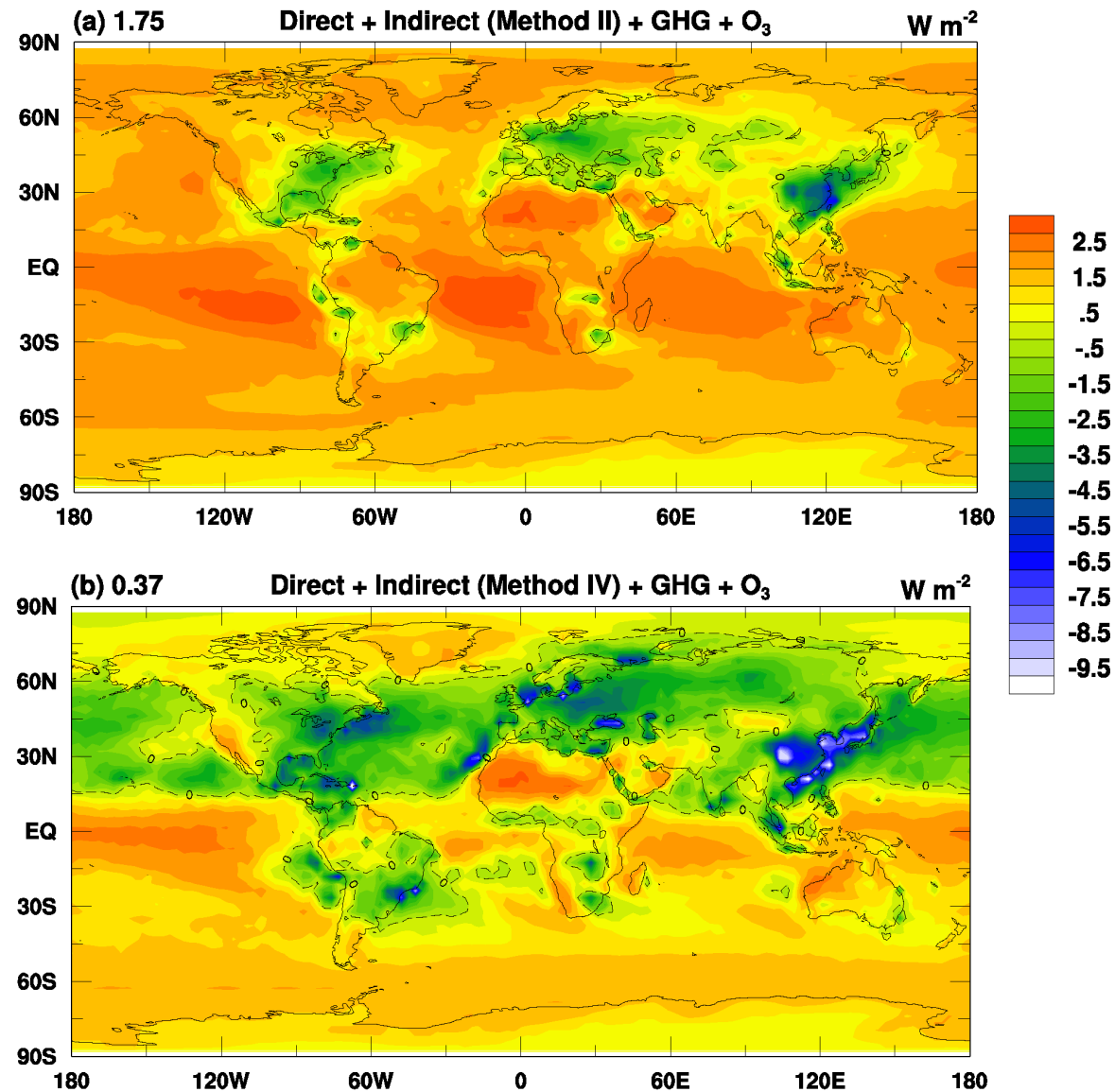


Boucher and Lohmann, 1995

SHORTWAVE FORCING, ANNUAL AVERAGE

GHG's + O₃ + Sulfate (Direct and Indirect)

Two Formulations of Cloud Droplet Concentration

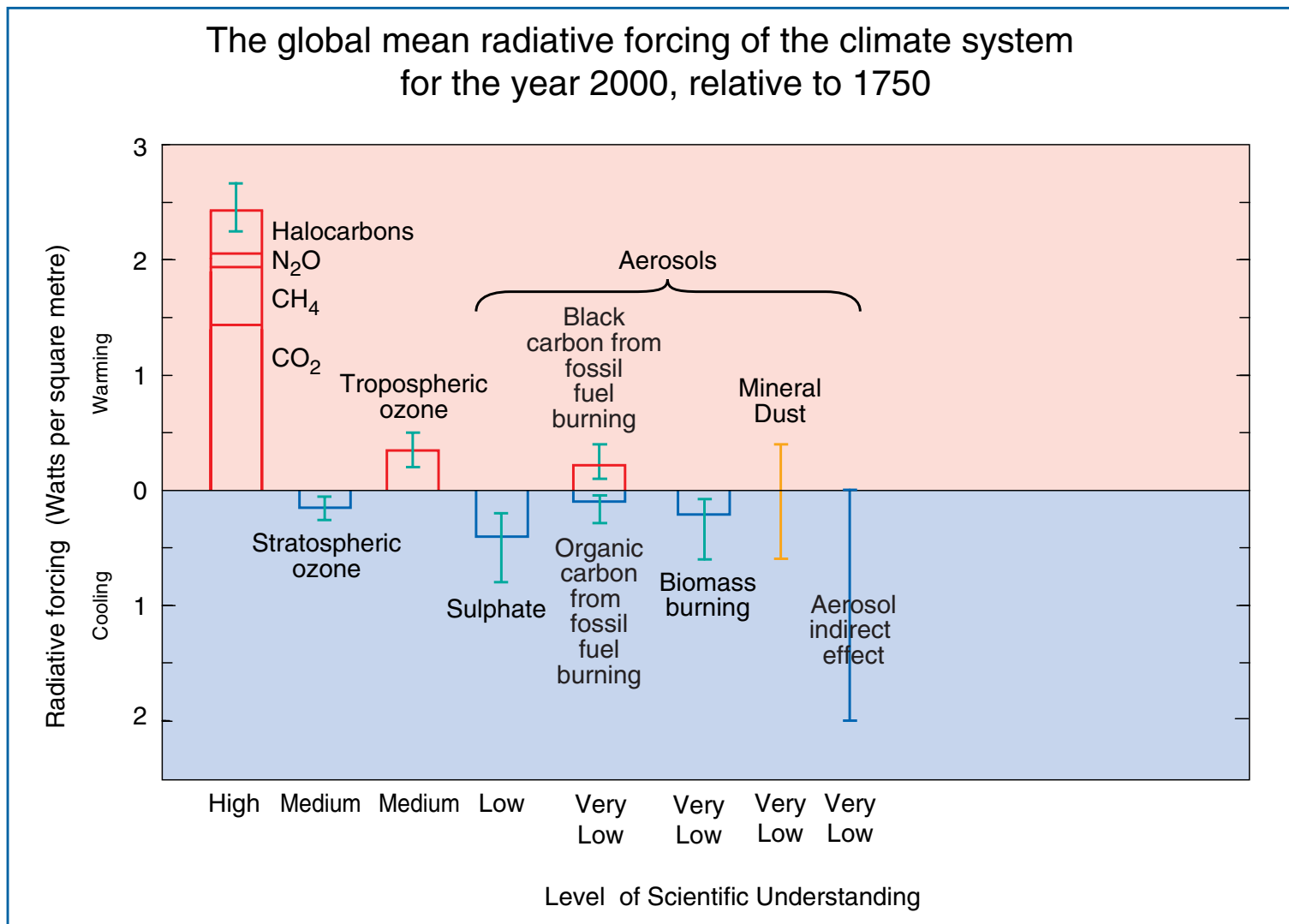


Kiehl et al., JGR, 2000

RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD

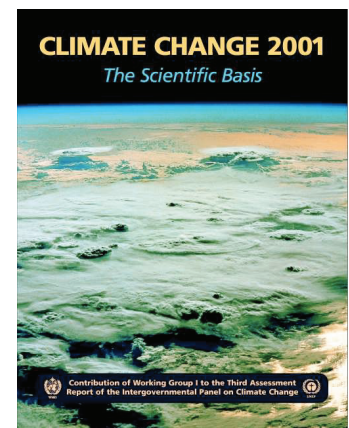
IPCC (2001)

GHG's and aerosol direct and indirect effects



Summary for Policymakers

A Report of Working Group I of the Intergovernmental Panel on Climate Change



WHY SO LARGE UNCERTAINTY IN AEROSOL FORCING?

- *Uncertainties in knowledge of atmospheric composition*

Mass loading and chemical and microphysical properties and cloud nucleating properties of anthropogenic aerosols, and geographical distribution.

At present and as a function of secular time.

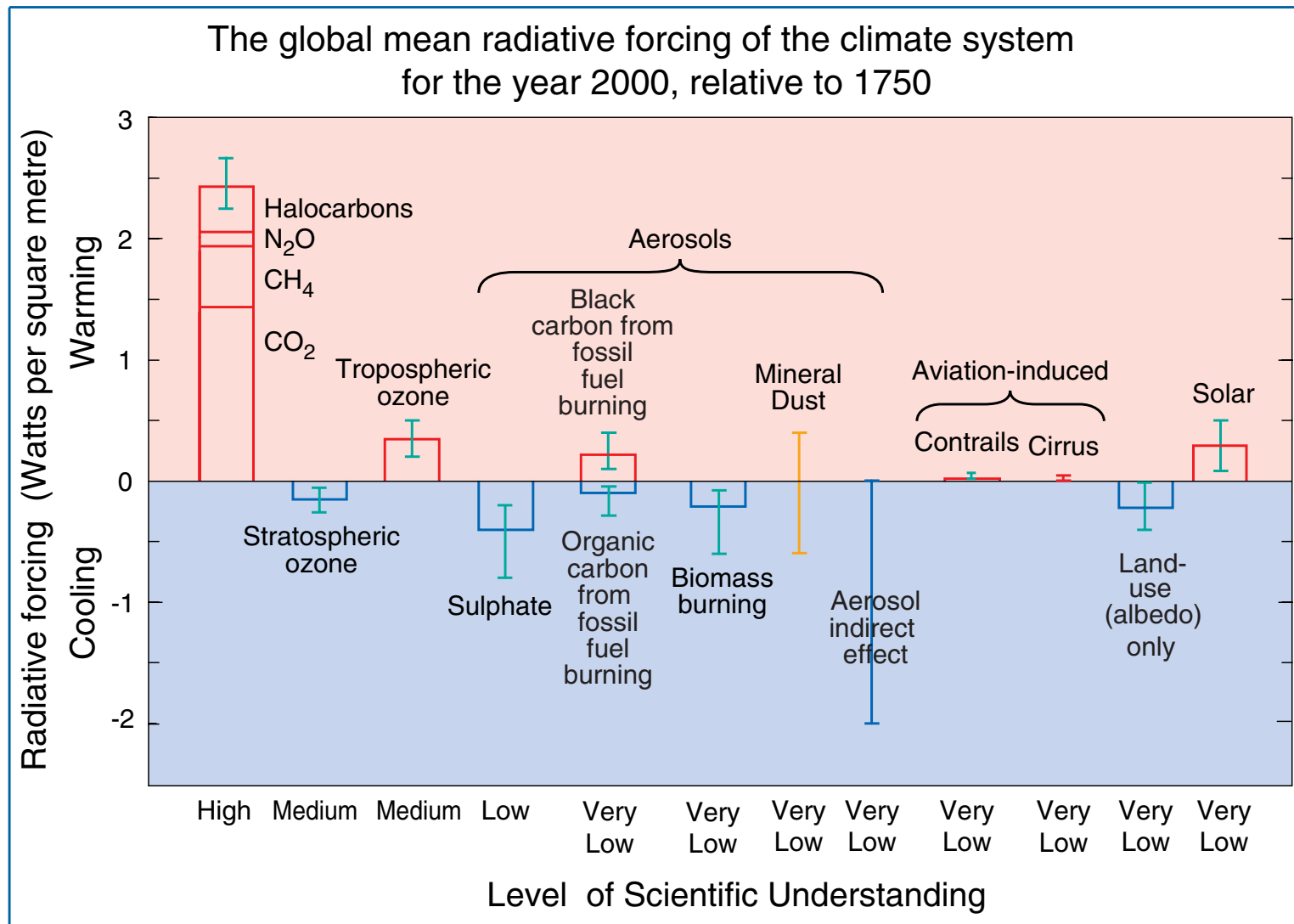
- *Uncertainties in knowledge of atmospheric physics of aerosols*

Relating direct radiative forcing and cloud modification by aerosols to their loading and their chemical and microphysical properties.

The Department of Energy is initiating a new research program examining aerosol chemistry and physics pertinent to radiative forcing of climate change.

RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD

IPCC (2001)



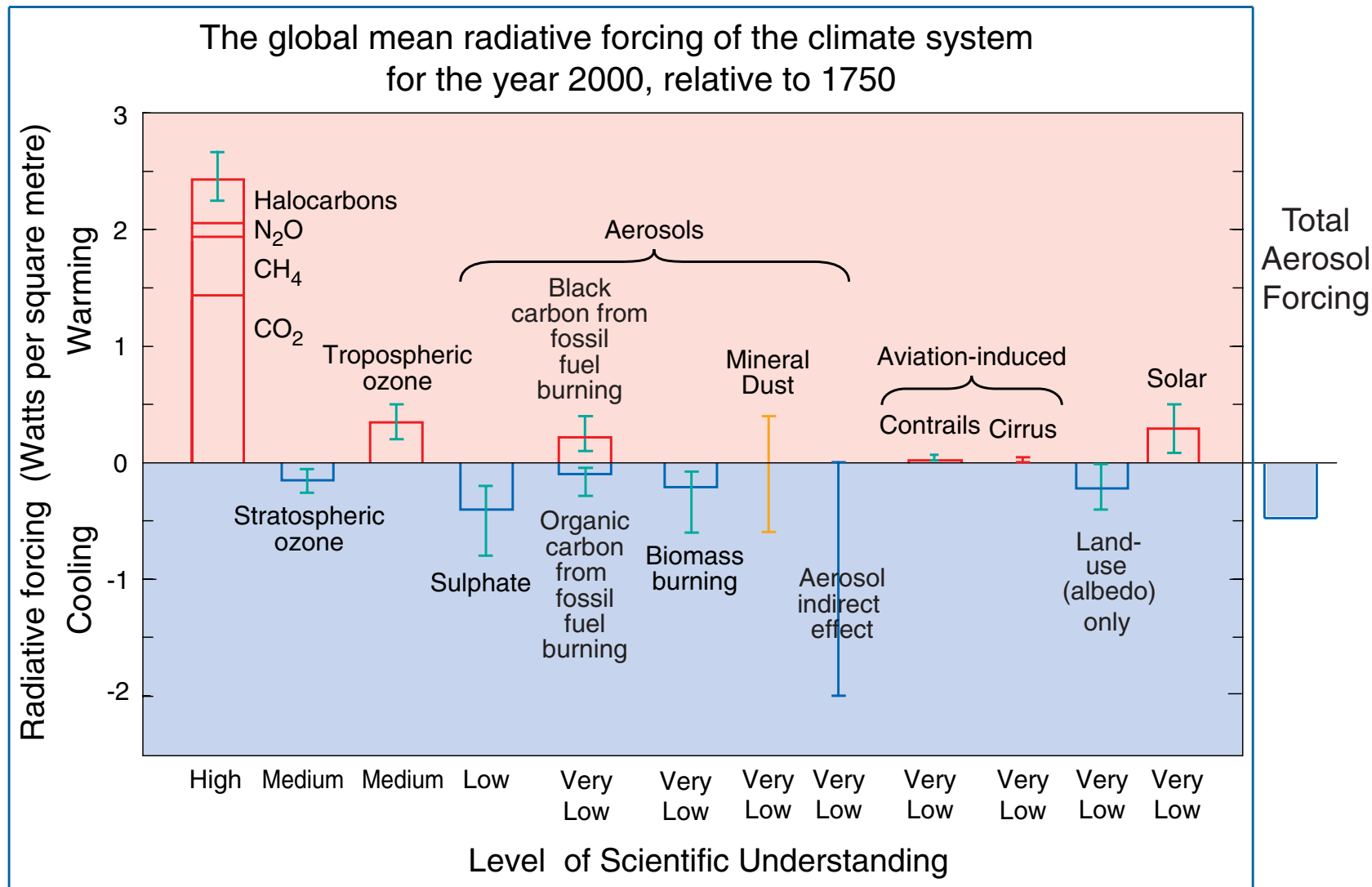
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RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD

IPCC (2001)

With total aerosol forcing



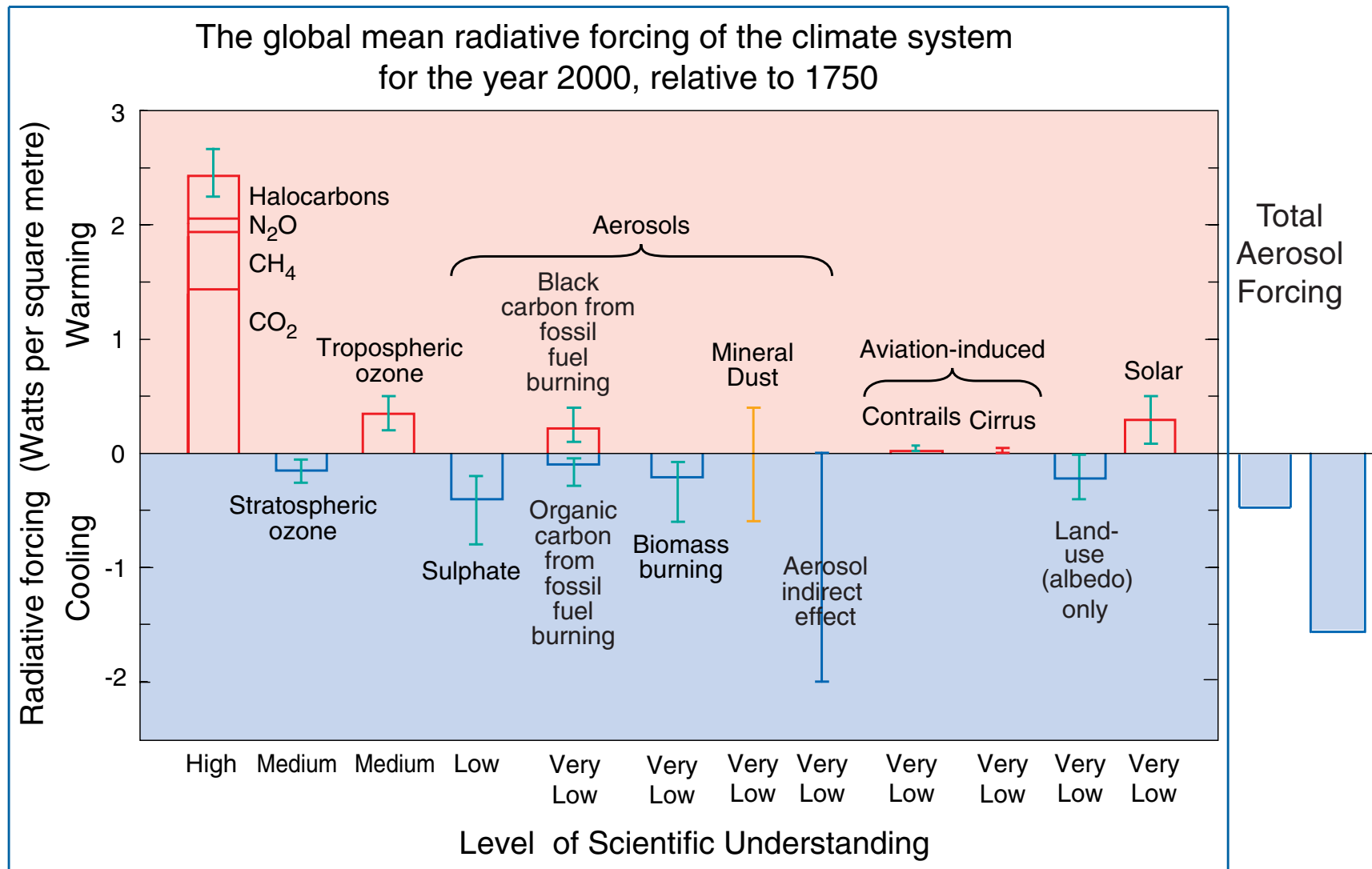
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RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD

IPCC (2001)

With total aerosol forcing



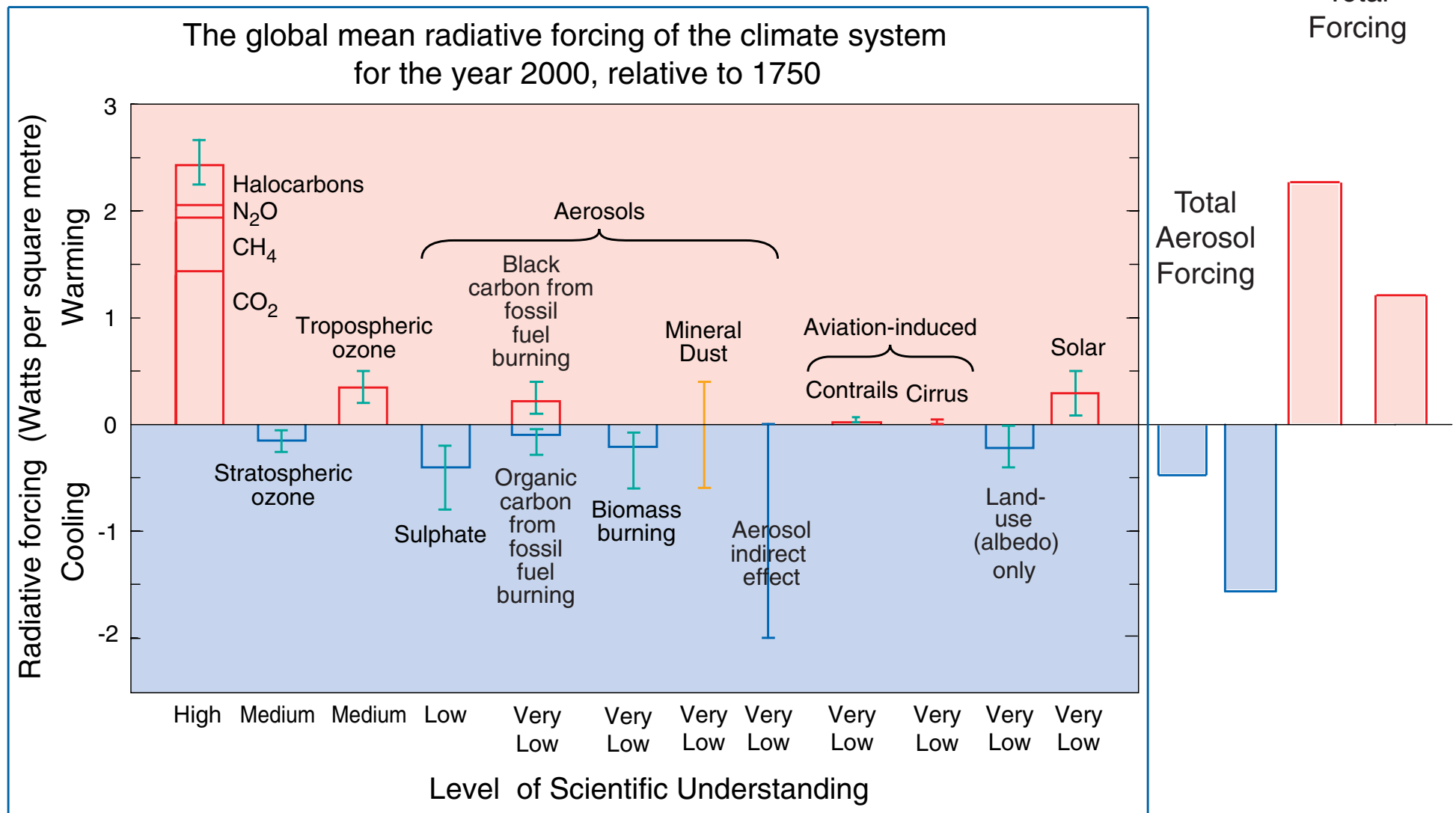
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RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD

IPCC (2001)

With total aerosol forcing and total forcing



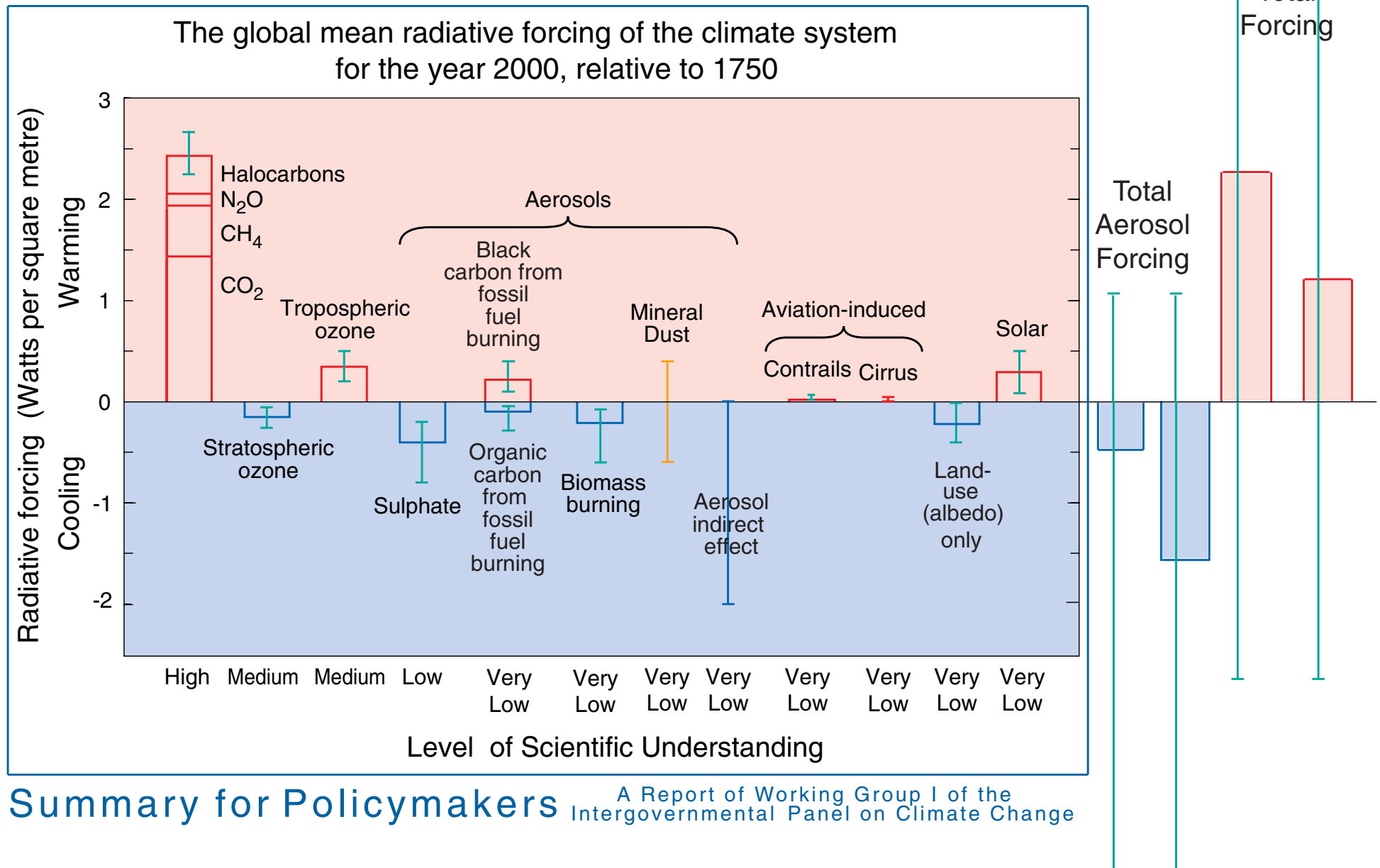
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RADIATIVE FORCING OVER THE INDUSTRIAL PERIOD

IPCC (2001)

With total aerosol forcing and total forcing and uncertainties



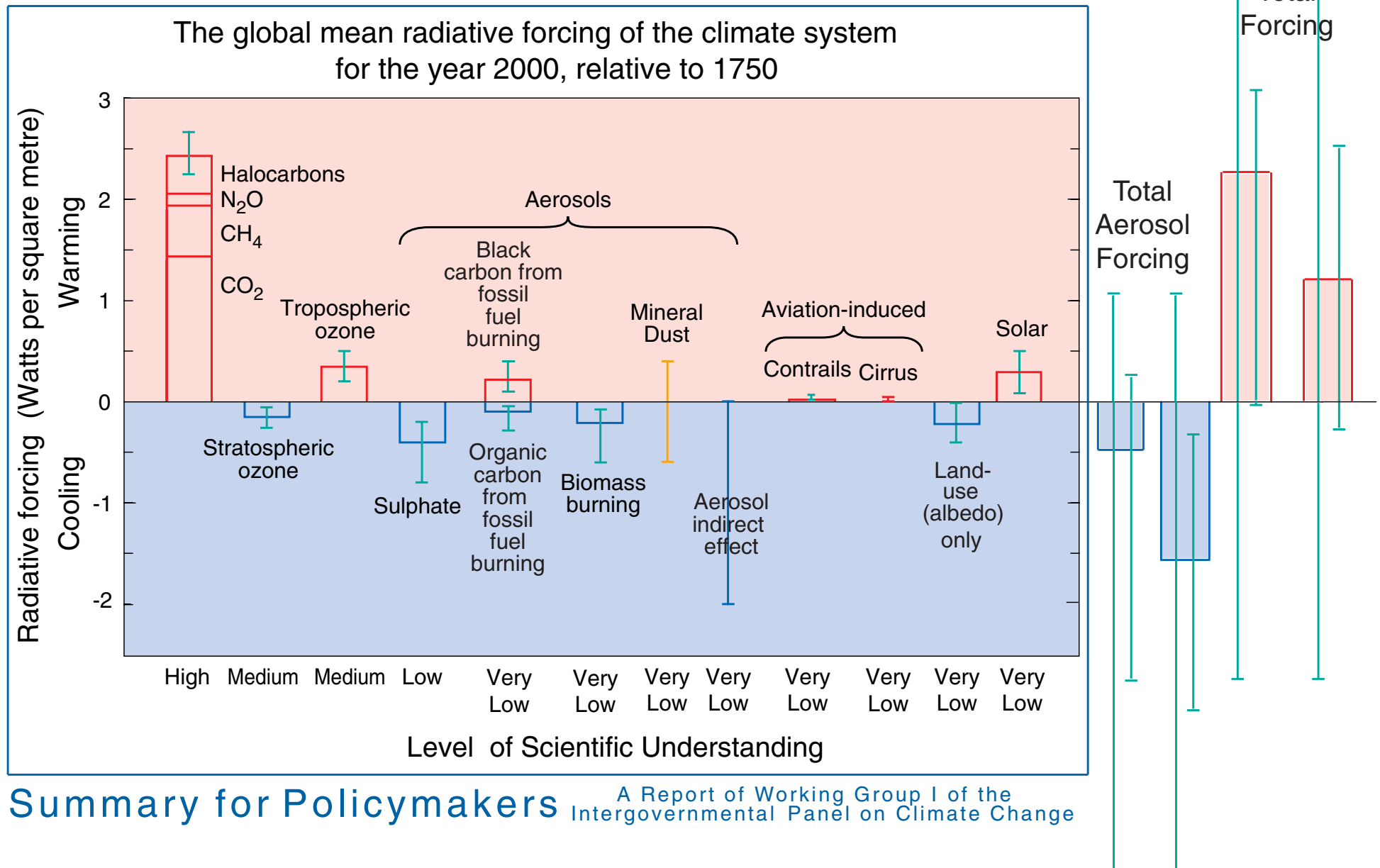
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IPCC (2001)

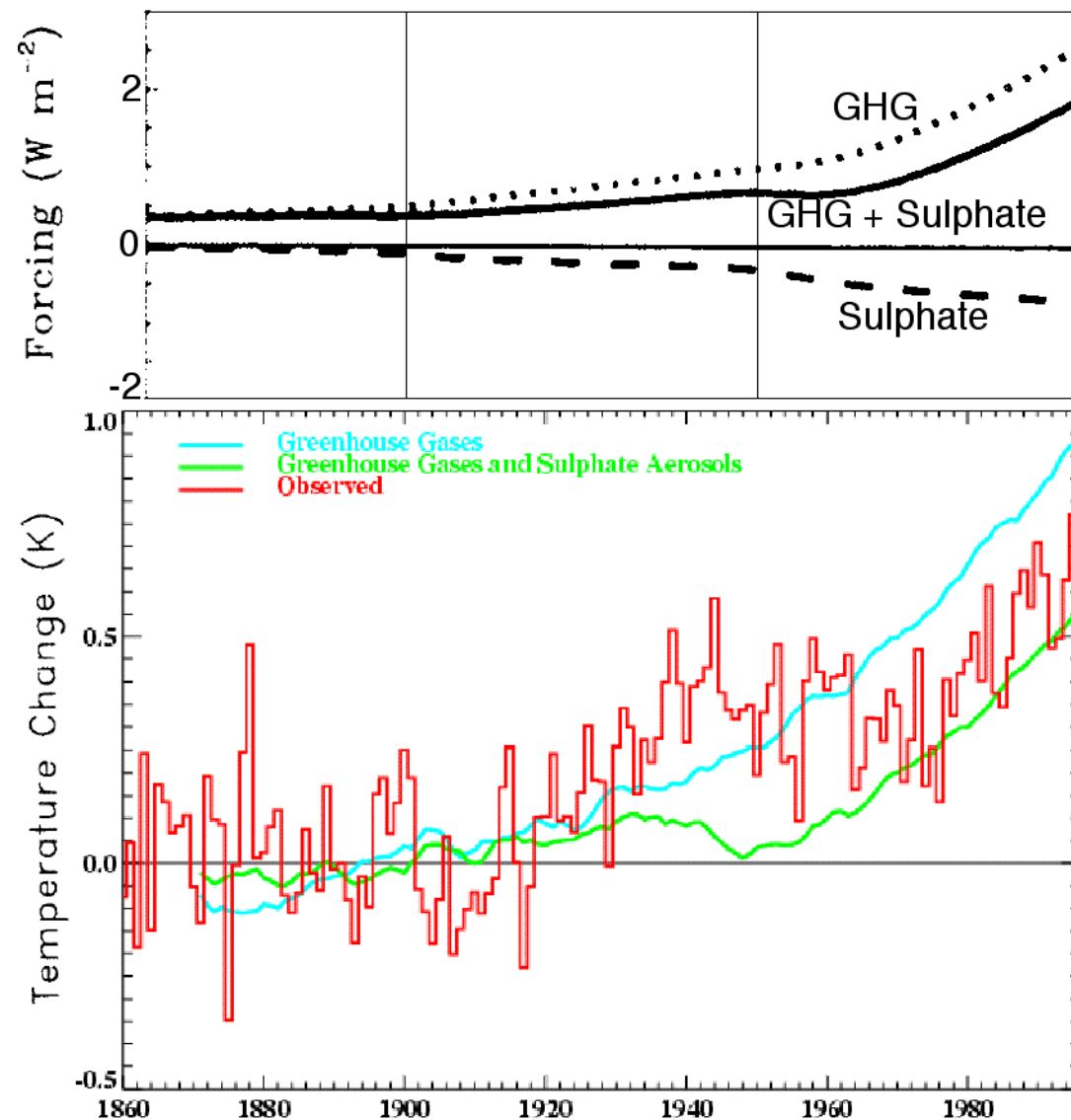
With total aerosol forcing and total forcing and uncertainties



REPRESENTING AEROSOL INFLUENCES IN CLIMATE MODELS

FORCING AND RESPONSE IN THE UK MET OFFICE MODEL (1995)

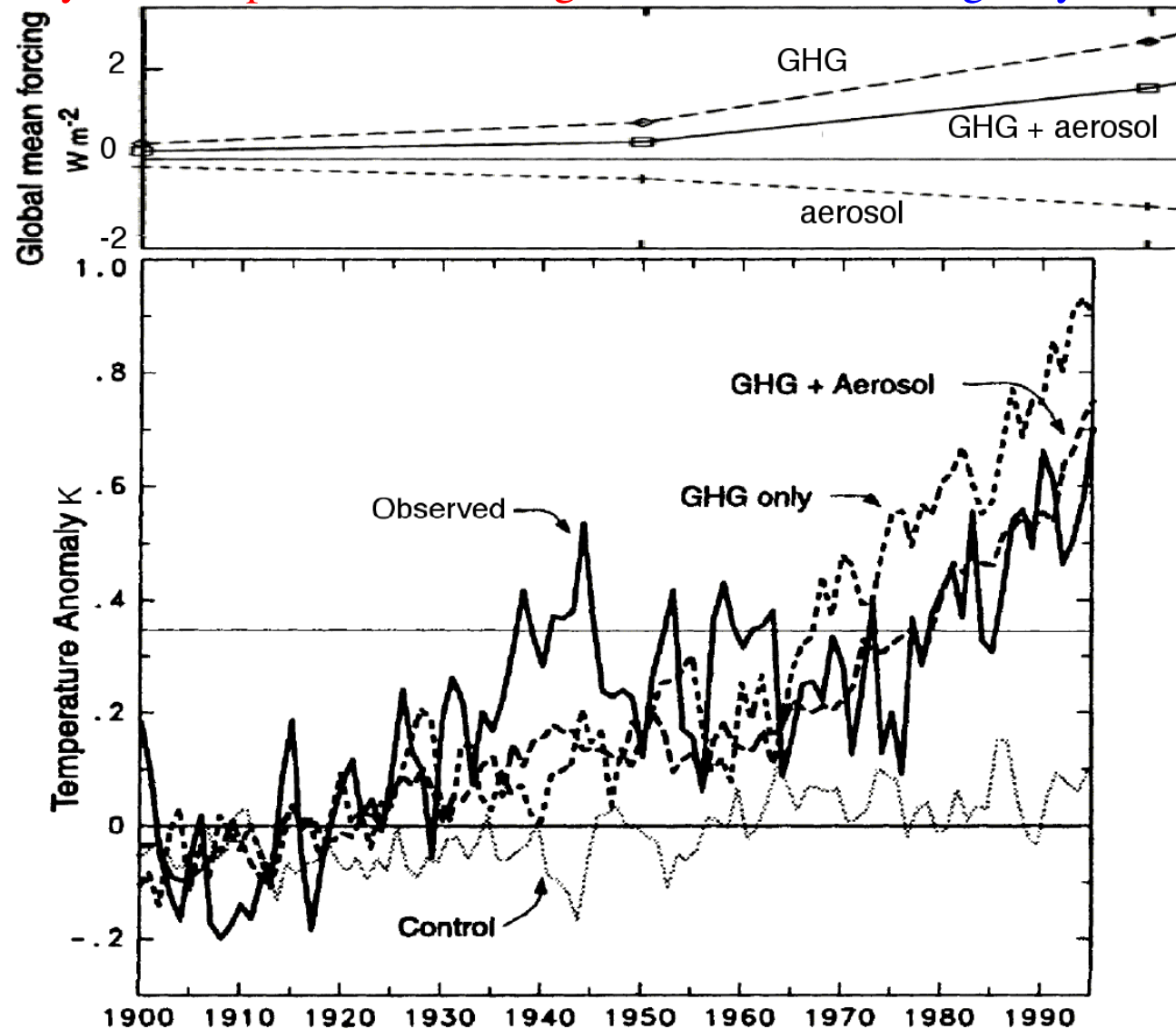
Model sensitivity = 2.5 K per CO₂ doubling; sulfate direct forcing only, -0.6 W m⁻² (1990)



“Inclusion of sulphate aerosol forcing *improves the simulation* of global mean temperature over the last few decades.” -- *Mitchell, Tett, et al., Nature, 1995*

FORCING AND RESPONSE IN THE CANADIAN CLIMATE MODEL (2000)

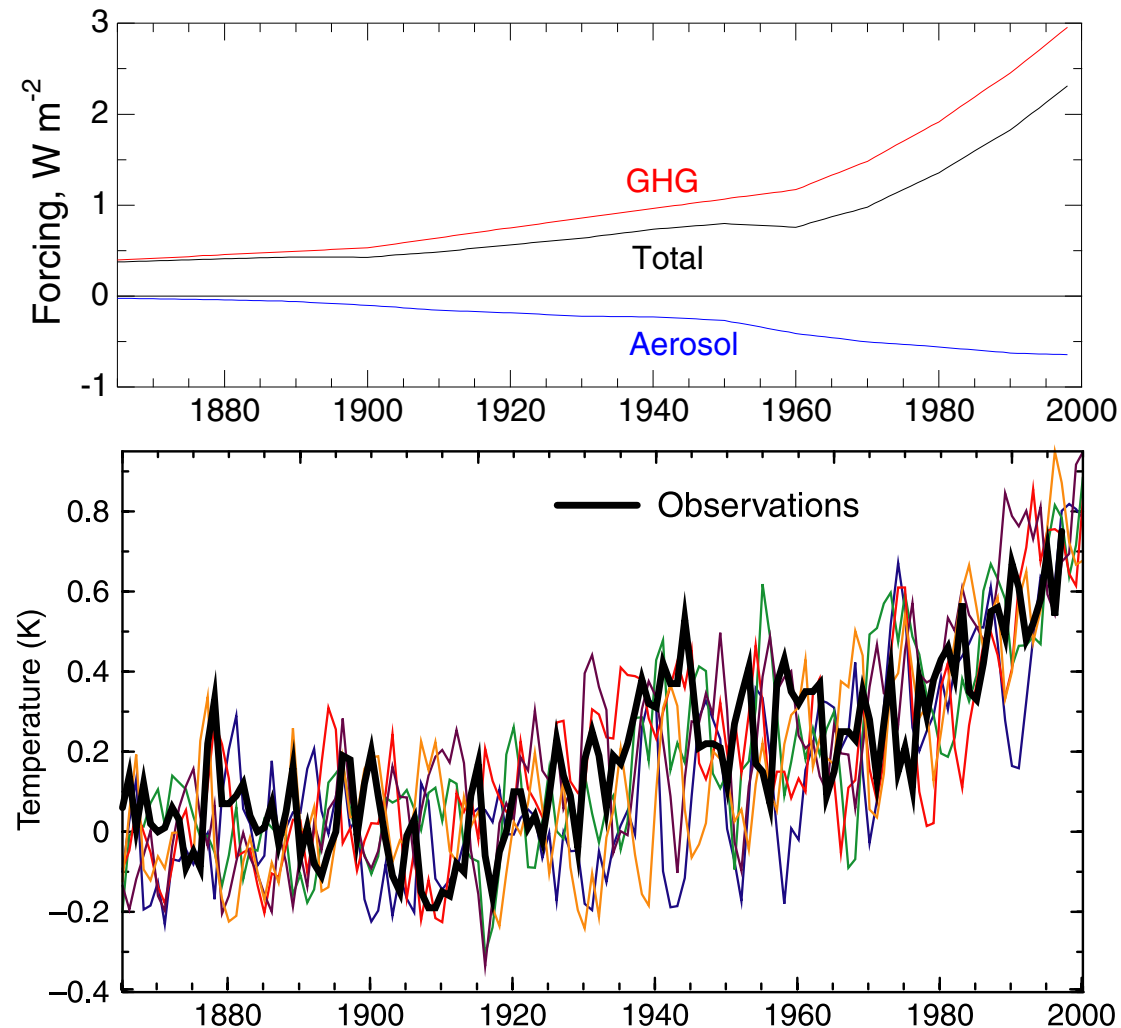
Model sensitivity = 3.5 K per CO₂ doubling; sulfate direct forcing only, -1.0 W m⁻² (1990)



“Observed global mean temperature changes and those simulated for GHG + aerosol forcing show *reasonable agreement*.” -- Boer, et al., *Climate Dynamics*, 2000

FORCING AND RESPONSE IN THE GFDL MODEL (2000)

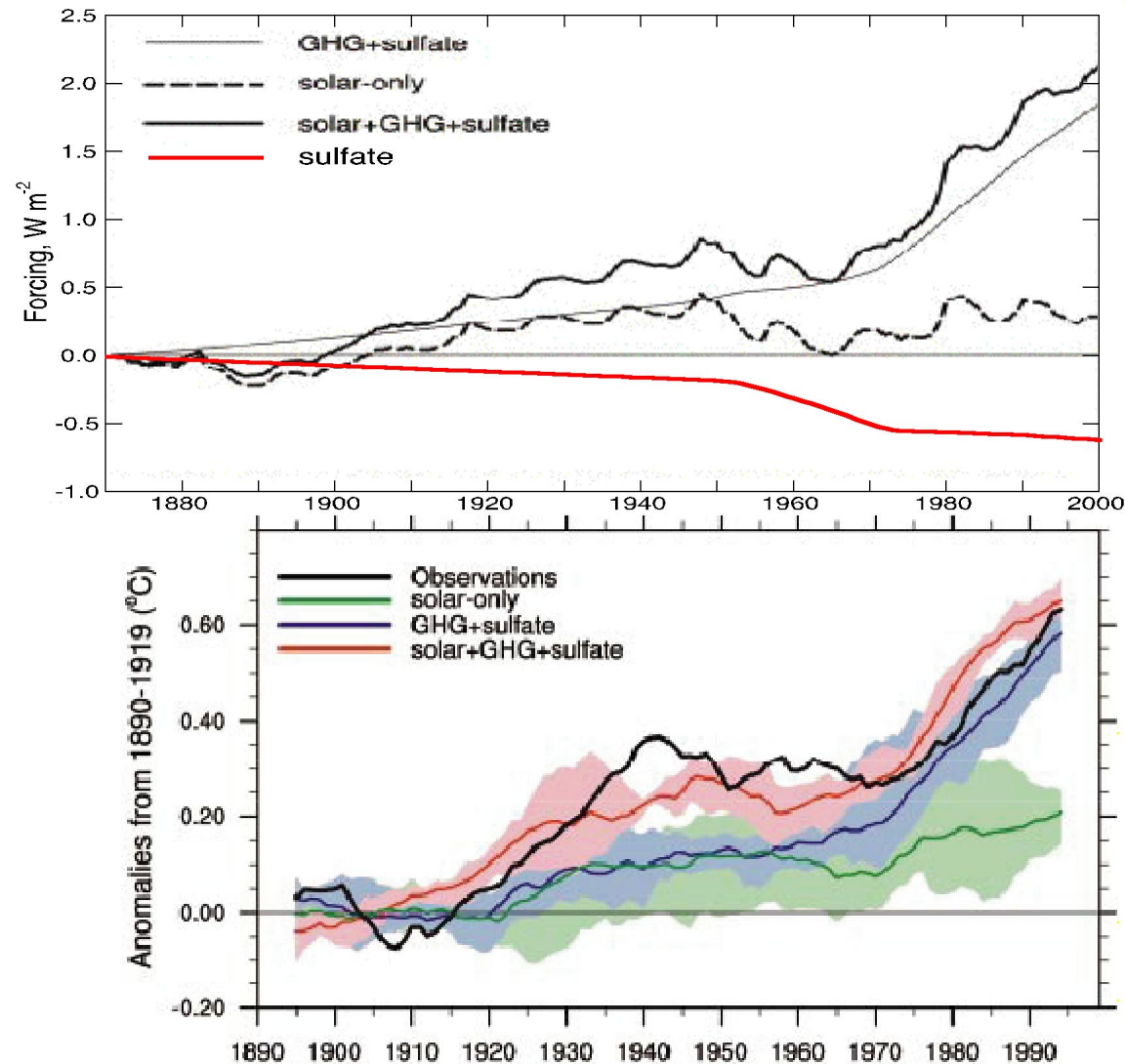
Model sensitivity = 3.4 K per CO₂ doubling; sulfate forcing, -0.62 W m⁻² (1990)



“The surface temperature time series from the five GHG-plus-sulfate integrations show an increase over the last century, which is *broadly consistent* with the observations.” -- *Delworth & Knutson, Science, 2000*

FORCING AND RESPONSE IN THE NCAR MODEL (2003)

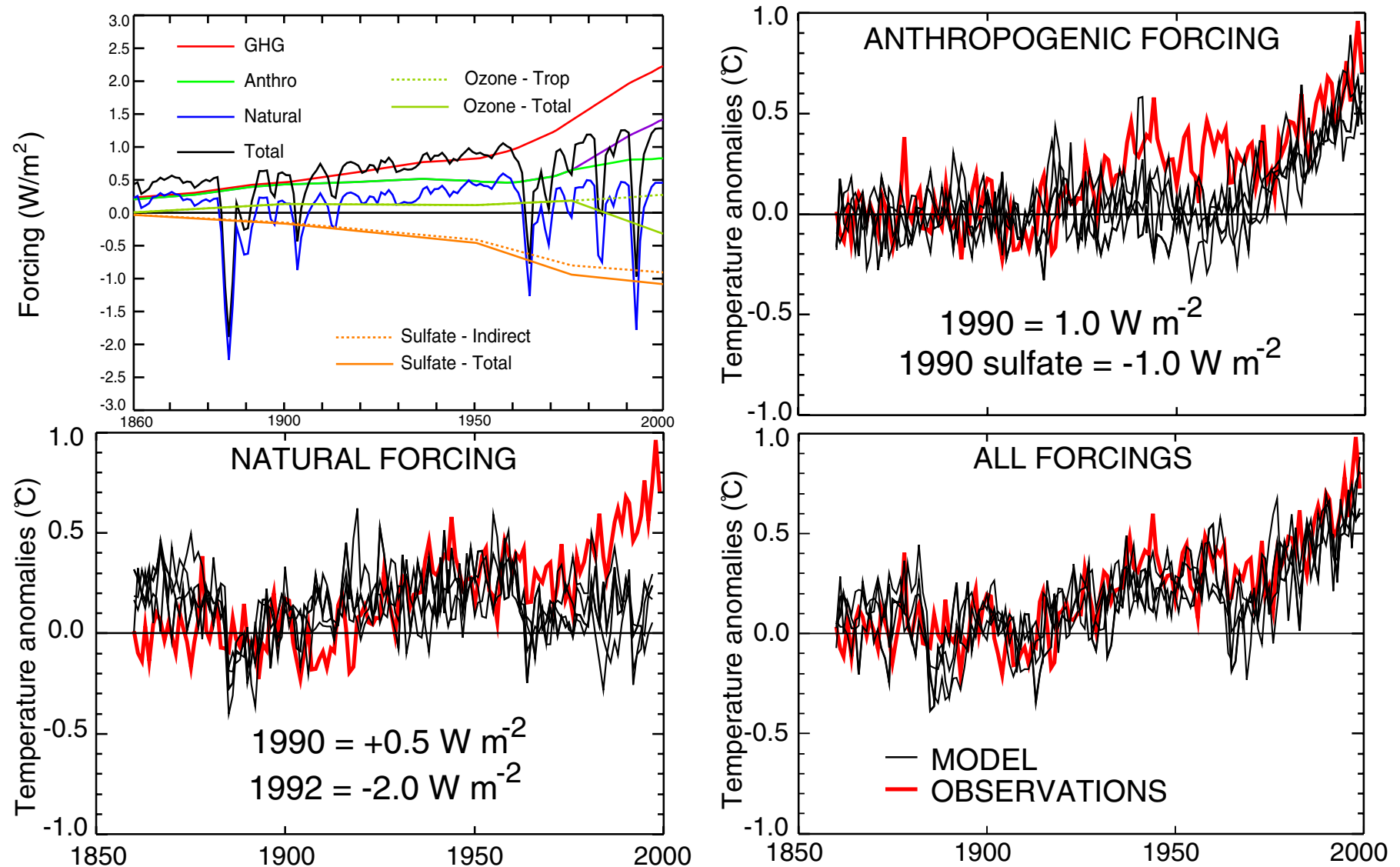
Model sensitivity = 2.18 K per CO₂ doubling; sulfate direct forcing only, -0.6 W m⁻² (1990)



“The time series from GHG + sulfates + solar shows *reasonable agreement* with the observations.” -- Meehl, Washington, Wigley et al., *J. Climate*, 2003.

FORCING AND RESPONSE IN THE UK MET OFFICE MODEL (2000)

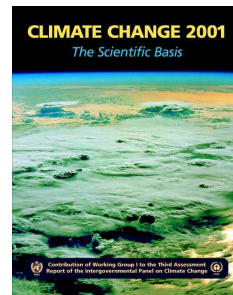
Model sensitivity = 3.45 K per CO₂ doubling; sulfate + indirect forcing, -1.1 W m⁻² (1990)



“The ALL ensemble *captures the main features* of global mean temperature changes observed since 1860.” -- Stott, Tett, Mitchell, et al., Science, 2000

IPCC-2001 STATEMENTS ON DETECTION AND ATTRIBUTION OF CLIMATE CHANGE

- “ *Simulations that include estimates of natural and anthropogenic forcing **reproduce the observed large-scale changes** in surface temperature over the 20th century.*
- “ *Most model estimates that take into account both greenhouse gases and sulphate aerosols are **consistent with observations** over this period.*



OUR SIMULATIONS THAT INCLUDE ESTIMATES
OF NATURAL AND ANTHROPOGENIC FORCING
REPRODUCE THE OBSERVED LARGE-SCALE
CHANGES IN SURFACE TEMPERATURE
OVER THE 20TH CENTURY.

BUT MOM, DON'T THE
GCM CALCULATIONS
REQUIRE ACCURATE
ESTIMATES OF
FORCING?

SHHHH!! THE EMPEROR
MIGHT HEAR YOU.



UNCERTAINTY PRINCIPLES

$$\text{Climate sensitivity } \lambda = \Delta T / F$$

The fractional uncertainty in climate sensitivity λ is evaluated from fractional uncertainties in temperature change ΔT and forcing F as:

$$\frac{\delta\lambda}{\lambda} = \sqrt{\left(\frac{\delta\Delta T}{\Delta T}\right)^2 + \left(\frac{\delta F}{F}\right)^2}$$

A reasonable target uncertainty might be:

$$\frac{\delta\lambda}{\lambda} = 30\%, \text{ e.g., } \Delta T_{2\times\text{CO}_2} = (3 \pm 1) \text{ K}$$

This would require uncertainties in temperature anomaly and forcing:

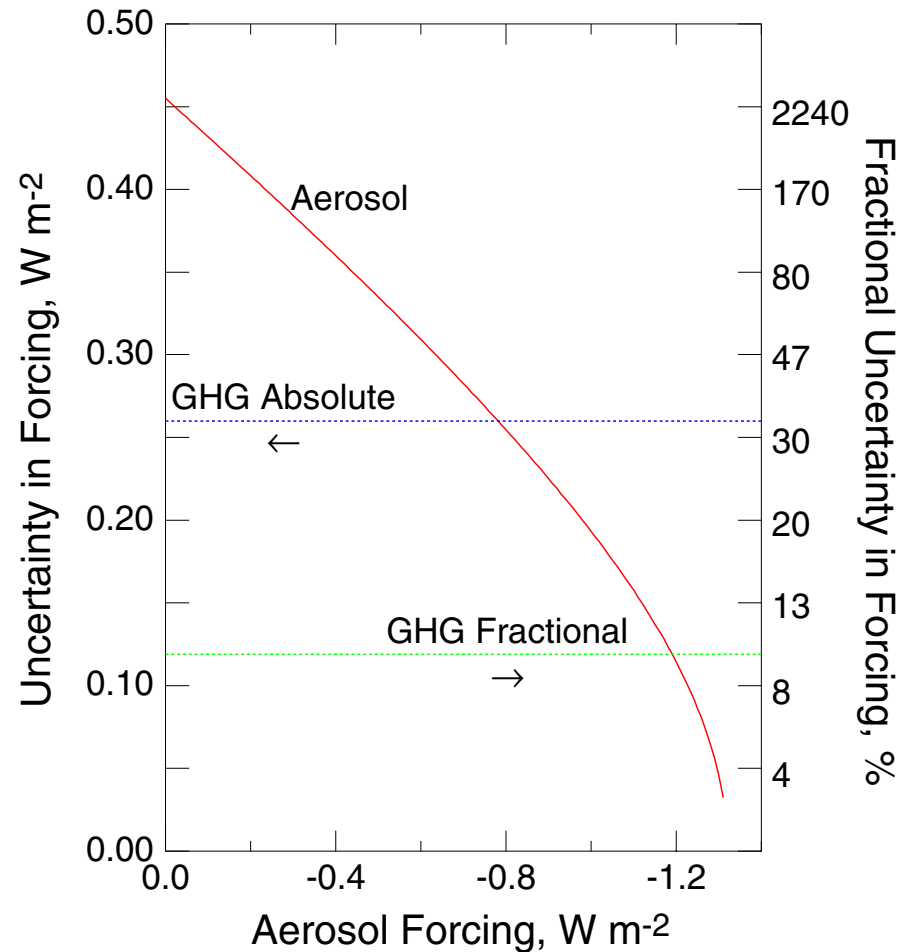
$$\frac{\delta\Delta T}{\Delta T} \approx \frac{\delta F}{F} \approx 20\%.$$

This imposes *stringent requirements on uncertainty in aerosol forcing!*

REQUIRED UNCERTAINTY IN AEROSOL FORCING

Uncertainty in total forcing not to exceed 20%

GHG Forcing (well mixed gases + strat and trop O₃) = $2.6 \text{ W m}^{-2} \pm 10\%$



Uncertainty in aerosol forcing must be reduced by at least a factor of 3 to meet requirements for determining climate sensitivity.

CONCLUSIONS

- *Radiative forcing of climate change by anthropogenic aerosols is substantial in the context of other forcings of climate change over the industrial period.*

Global annual mean aerosol forcing of -1 to -3 W m^{-2} is plausible given present understanding.

- *Uncertainty in radiative forcing of climate change by anthropogenic aerosols is the **greatest source of uncertainty** in forcing of climate change.*

This uncertainty precludes:

- ***Evaluation of models** of climate change.*
- ***Inference of climate sensitivity** from temperature changes over the industrial period.*
- ***Informed policy making** on greenhouse gases.*
- *Uncertainty in aerosol forcing must be reduced **at least three-fold** for uncertainty in climate sensitivity to be meaningfully reduced and bounded.*

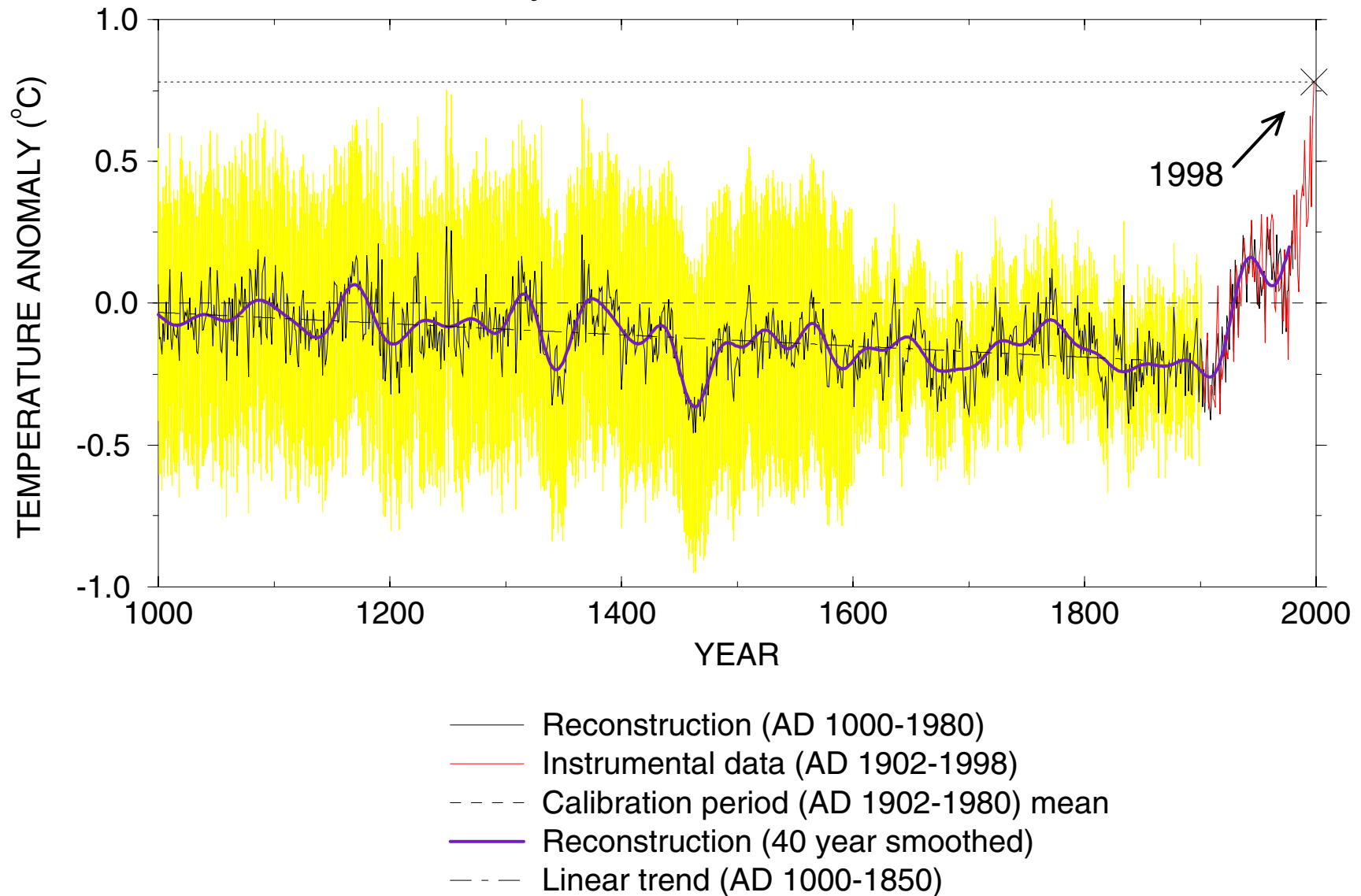
SOME CONCLUDING OBSERVATIONS

- GHG concentrations and forcing are increasing. GHGs persist in the atmosphere for decades to centuries.
- Aerosol forcing is comparable to greenhouse gas forcing but much more uncertain.
- Hence total forcing over the industrial period is highly uncertain.
- Hence the sensitivity of the climate system remains highly uncertain.
- Climate sensitivity will remain uncertain unless and until aerosol uncertainty is substantially decreased.
- Decisions must be made in an uncertain world. (Lack of controls on GHG emissions is also a decision).

NORTHERN HEMISPHERE TEMPERATURE TREND (1000-1998)

From tree-ring, coral, and ice-core proxy records

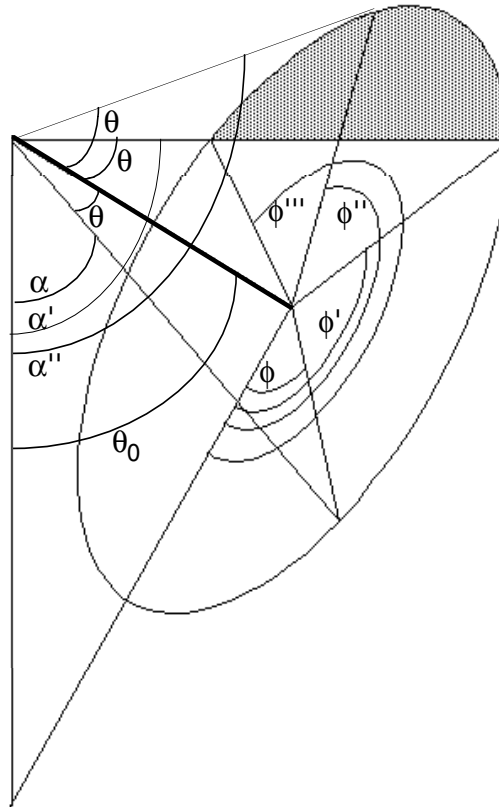
As calibrated by instrumental measurements



Mann et al., GRL, 1999

UPSCATTER FRACTION

SCATTERING OF SOLAR RADIATION BY AEROSOL PARTICLE



Upscatter fraction β is the fraction of radiation scattered into the upward hemisphere.

$$\beta = \frac{\int_{\text{upward hemisphere}} P(\theta, \phi) d\Omega}{\int_{4\pi} P(\theta, \phi) d\Omega} = \frac{\int_{\cos\alpha < 0} P(\theta, \phi) d\Omega}{4\pi}$$